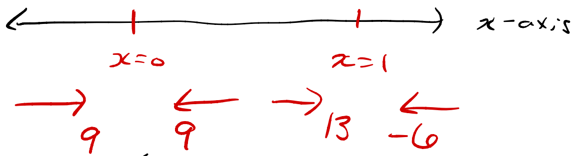
Lecture 5 : The derivative

$\boxed{\text{HW 4 \#11}}$ $f(x) = \begin{cases} 7x^2 + 9 & x \le 0 \\ 4x + 9 & 0 < x < 1 \\ -4x - 2 & x \geq 1 \end{cases}$

find / classify discont



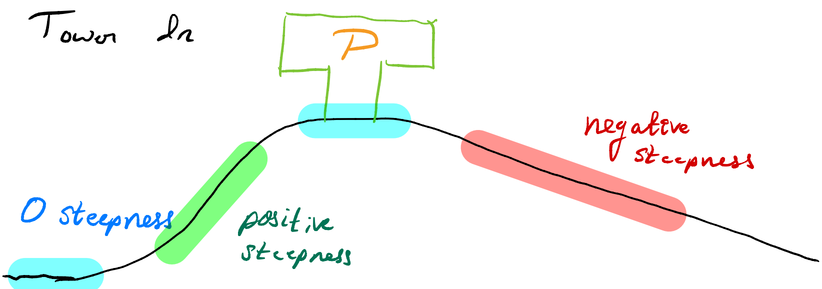
$x = 0$: $\lim\limits_{x \to 0} f(x) = 9$ ✓ , $f(0) = 7(0)^2 + 9 = 9$ ✓

$\lim\limits_{x \to 0} f(x) = f(0)$ ✓

f is cont at $x = 0$

$x = 1$: $\lim\limits_{x \to 1} f(x)$ DNE ✗ discont. jump

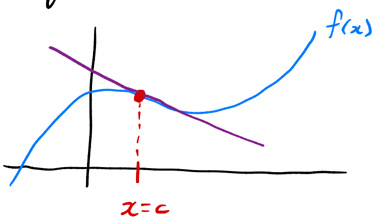
Tower Dr



Q: How steep is the hill? (rate of change)

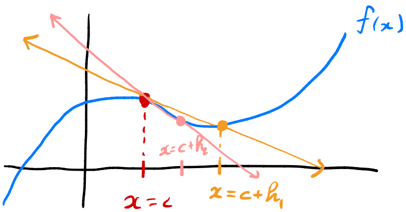
Derivative of $f(x)$ at $x = c$ is the steepness of f at the value $x = c$. $f'(c)$ or $\frac{d}{dx}[f]\Big|_{x=c}$

Tangent lines to f



tangent line to f at $x=c$ is characterized by

- passes through pt $(c, f(c))$
- it has the steepness as f at $x=c$



Definition of the derivative

$$f'(c) = \left(\frac{df}{dx}\Big|_c = \frac{d}{dx}[f]\Big|_{x=c}\right) = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

$$\star\ f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \text{slope (steepness) of } f \text{ at } x$$

$$= \text{slope of the tangent line at } x \quad \star$$

e.g. ① $f(x) = 3/(6x-5)$; finding $f'(x)$

$$f'(x) = \lim_{h\to 0}\left(\frac{\dfrac{3}{(6(x+h)-5)} - \dfrac{3}{6x-5}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\dfrac{3(6x-5) - 3(6x+6h-5)}{(6x+6h-5)(6x-5)}\cdot\dfrac{1}{h}}{h\cdot 1/h}\right)$$

$$= \lim_{h\to 0}\frac{18x-15-18x-18h+15}{h(6x+6h-5)(6x-5)}$$

$$= \lim_{h\to 0}\frac{-18h}{h(6x+6h-5)(6x-5)}$$

$$= \lim_{h \to 0} \frac{-18}{(6x+6h-5)(6x-5)} = \frac{-18}{(6x+6\cdot 0-5)(6x-5)}$$

$$= \frac{-18}{(6x-5)^2}$$

② find the tangent line to $f(x) = -2x^2 - 8$ at $x = 0$.

Point-slope form of a line:

$$y - y_1 = m(x - x_1)$$

- (x_1, y_1) is a pt on the line
- m is the slope

$$(x_1, y_1) = (c, f(c)) \quad \text{and} \quad m = f'(c)$$
$$= (0, f(0))$$
$$= (0, -8)$$

$$m = f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$
$$= \lim_{h \to 0} \frac{-2(0+h)^2 - 8 - [-2(0)^2 - 8]}{h}$$
$$= \lim_{h \to 0} \frac{-2h^2 - 8 + 8}{h}$$
$$= \lim_{h \to 0} \frac{-2h^2}{h}$$
$$= \lim_{h \to 0} -2h$$
$$= -2 \cdot 0 = 0$$

Equation for the tan line: $\overset{y_1}{y+8} = \overset{m}{0}(x - \overset{x_1}{0}) \rightsquigarrow y = -8$

Lecture 5: The derivative.

HW 4 # 9 $f(x) = \begin{cases} x - 12 & x < 16 \\ \sqrt{x} & x \geq 16 \end{cases}$

find / classify all discont.

$x = 16$?

1) $f(16) = \sqrt{16} = 4$ ✓

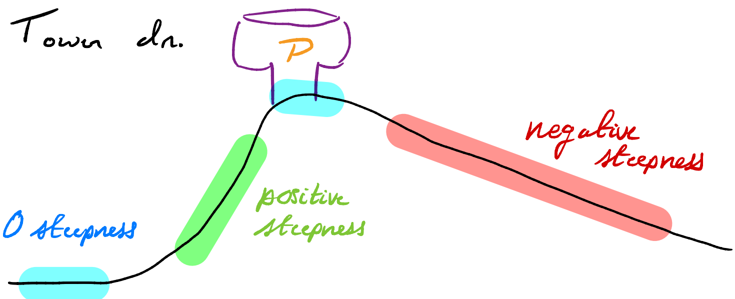
2) $\lim\limits_{x \to 16^-} f(x) = 16 - 12 = 4$ $\qquad \lim\limits_{x \to 16^+} f(x) = \sqrt{16} = 4$

$\lim\limits_{x \to 16} f(x) = 4$ ✓

3) $f(16) = 4 = \lim\limits_{x \to 16} f(x)$ ✓

So f is cont. at $x = 16$.
No discont. !

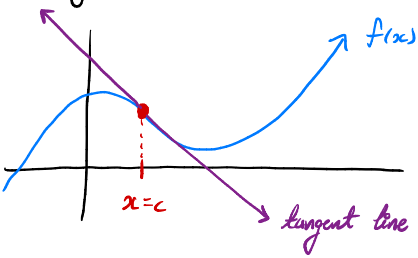
Town dr.



Derivative of $f(x)$ at $x = c$ is the steepness (• slope • rate of change) of f at the value $x = c$. We denote this by

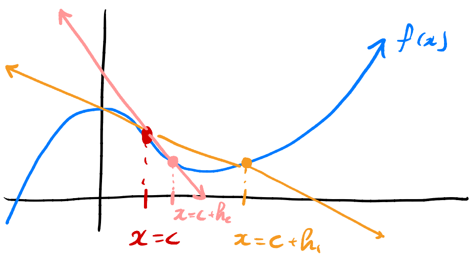
$$f'(c) = \frac{df}{dx}\bigg|_{x=c} = \frac{d}{dx}[f]\bigg|_{x=c}$$

Tangent lines to f



tangent line to f at $x=c$ is characterized by

- passes through the pt $(c, f(c))$
- it has the same steepness (slope) as f at $x=c$.



Definition of the derivative

$$f'(c) \left(= \frac{df}{dx}\Big|_{x=c} = \frac{d}{dx}[f]\Big|_{x=c}\right)$$

$$= \lim_{h\to 0} \underbrace{\frac{f(c+h)-f(c)}{h}}_{\text{slope of approximation of tan. line}}$$

$$* \quad f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

THE DERIVATIVE

is the function which outputs the slope (steepness) of f at x. $*$

e.g. $f(x) = 3/(6x-5)$; find $f'(x)$

$$f'(x) = \lim_{h\to 0}\left(\frac{\dfrac{3}{(6(x+h)-5)} - \dfrac{3}{(6x-5)}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\dfrac{3(6x-5)-3(6x+6h-5)}{(6x+6h-5)(6x-5)}\cdot\dfrac{1}{h}}{h\cdot\dfrac{1}{h}}\right)$$

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$$= \lim_{h\to 0}\frac{-18h}{h(6x+6h-5)(6x-5)}$$

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$$= \frac{-18}{(6x-5)^2}$$

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$$(x_1, y_1) = (c, f(c)) = (0, f(0)) = (0, -8)$$

$$m = f'(c) = f'(0)$$

$$m = f'(0) = \lim_{h \to 0} \frac{f(\overset{c}{0} + h) - f(\overset{c}{0})}{h}$$

$$= \lim_{h \to 0} \frac{-2(0+h)^2 - 8 - [-2(0)^2 - 8]}{h}$$

$$= \lim_{h \to 0} \frac{-2h^2 - 8 + 8}{h}$$

$$= \lim_{h \to 0} \frac{-2h^2}{h}$$

$$= \lim_{h \to 0} -2h$$

$$= -2(0)$$

$$= 0$$

$$y - y_1 = m(x - x_1)$$

$$y + 8 = 0(x - 0)$$

$$y = -8 \longleftarrow$$ Equation for the tangent line to f at $x = 0$.