# Lecture 6: Basic rules of differentiation; Derivative of sin, cos, and natural exponential.

The derivative of $f(x)$ is

$$f'(x) = \frac{d}{dx}[f] = \frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

**Derivative of a constant**

if $c$ is a number, then $\frac{d}{dx}[c] = 0$

**Power Rule**

let $n$ be a nonzero number, then

$$\frac{d}{dx}[x^n] = n x^{n-1}$$

says $\frac{d}{dx}$ is linear

**Constant multiple**

$$\frac{d}{dx}[c f(x)] = c \cdot \frac{d}{dx}[f(x)]$$

**Sum Rule**

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

e.g. ① $y = x^8 + 3x^4 + 2$; find $y'$

$$y' = \frac{d}{dx}[y] = \frac{d}{dx}[x^8 + 3x^4 + 2]$$ sum rule

$$= \frac{d}{dx}[x^8] + \frac{d}{dx}[3x^4] + \frac{d}{dx}[2]$$

$$= 8x^7 + 3\frac{d}{dx}[x^4] + 0$$

$$= 8x^7 + 3 \cdot 4x^3 + 0$$

$$= 8x^7 + 12x^3$$

② $y = 8\sqrt{x} - \frac{2}{x^2} + 3x^{\pi-1} + 100$; find $y'$

$$y = 8x^{1/2} - 2x^{-2} + 3x^{\pi-1} + 100$$

$$y' = \frac{d}{dx}[8x^{1/2} - 2x^{-2} + 3x^{\pi-1} + 100]$$

$$= 8\frac{d}{dx}[x^{1/2}] - 2\frac{d}{dx}[x^{-2}] + 3\frac{d}{dx}[x^{\pi-1}] + \frac{d}{dx}(100]$$

$$= 8 \cdot \frac{1}{2}x^{1/2-1} - 2(-2)x^{-2-1} + 3(\pi-1)x^{\pi-1-1} + 0$$

$$= 4x^{-1/2} + 4x^{-3} + 3(\pi-1)x^{\pi-2}$$

## Derivative of sin / cos

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

e.g. 3 $f(x) = 5\cos x - 3\sin x$; find $f'(x)$

$$f'(x) = \frac{d}{dx}[5\cos x - 3\sin x]$$

$$= 5\frac{d}{dx}[\cos x] - 3\frac{d}{dx}[\sin x]$$

$$= 5(-\sin x) - 3\cos x$$

$$= -5\sin x - 3\cos x$$

## Derivative of natural exponential

$$\frac{d}{dx}[e^x] = e^x$$

e.g. 4 $g(x) = \frac{10x + \sqrt{x}}{x^2}$ ; $g'(x)$

$$= \frac{10x}{x^2} + \frac{x^{1/2}}{x^2}$$

$$= 10x^{-1} + x^{1/2-2}$$

$$= 10x^{-1} + x^{-3/2}$$

$$g'(x) = 10\frac{d}{dx}[x^{-1}] + \frac{d}{dx}[x^{-3/2}]$$

$$= 10(-1)x^{-2} - \frac{3}{2}x^{-3/2-1}$$

$$= -10x^{-2} - \frac{3}{2}x^{-5/2}$$

5 $y = -5e^{x} + \sin x$ ; $y'$

$$y = \frac{d}{dx}\left( -5e^{x} + \sin x \right]$$

$$= -5\frac{d}{dx}[e^{x}] + \frac{d}{dx}[\sin x]$$

$$= -5e^{x} + \cos x$$

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The derivative of $f(x)$ is

$$f'(x) = \frac{d}{dx}[f] = \frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

## Constant rule

if $c$ is a number $\frac{d}{dx}[c] = 0$

## Power rule

let $n$ be a nonzero number

$$\frac{d}{dx}[x^n] = n x^{n-1}$$

linearity of the derivative

$\frac{d}{dx}$ is a linear.

## Constant multiple rule

$$\frac{d}{dx}[c f(x)] = c \frac{d}{dx}[f(x)]$$

## Sum rule

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$$

e.g. ① $y = x^8 + 3x^4 + 2$; find $y'$

$$y' = \frac{d}{dx}[y] = \frac{d}{dx}[x^8 + 3x^4 + 2]$$

$$= \frac{d}{dx}[x^8] + \frac{d}{dx}[3x^4] + \frac{d}{dx}[2]$$

$$= \frac{d}{dx}[x^8] + 3\frac{d}{dx}[x^4] + \frac{d}{dx}[2]$$

$$= 8x^{8-1} + 3 \cdot 4x^{4-1} + 0$$

$$= 8x^7 + 12x^3$$

② $y = 8\sqrt{x} - \frac{2}{x^2} + 3x^{\pi-1} + 100$; find $y'$

$$y = 8x^{1/2} - 2x^{-2} + 3x^{\pi-1} + 100$$

$$y' = \frac{d}{dx}[8x^{1/2} - 2x^{-2} + 3x^{\pi-1} + 100]$$

$$= 8\frac{d}{dx}[x^{1/2}] - 2\frac{d}{dx}[x^{-2}] + 3\frac{d}{dx}[x^{\pi-1}] + \frac{d}{dx}[100]$$

$$= 8 \cdot \frac{1}{2} x^{\frac{1}{2}-1} - 2(-2)x^{-2-1} + 3(\pi-1)x^{\pi-1-1} + 0$$

$$= 4x^{-1/2} + 4x^{-3} + 3(\pi-1)x^{\pi-2}$$

## Derivative of sin / cos

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

e.g. 3 $f(x) = 5\cos x - 3\sin x$; find $f'(x)$

$$f'(x) = \frac{d}{dx}[5\cos x - 3\sin x]$$

$$= \frac{d}{dx}[5\cos x] + \frac{d}{dx}[-3\sin x]$$

$$= 5\frac{d}{dx}[\cos x] - 3\frac{d}{dx}[\sin x]$$

$$= 5(-\sin x) - 3(\cos x)$$

$$= -5\sin x - 3\cos x$$

## Derivative of the natural exponential

$$\frac{d}{dx}[e^x] = e^x$$

e.g. 4 $y = -5e^x + \sin x$; find $y'$

$$y' = \frac{d}{dx}[-5e^x + \sin x]$$

$$= -5\frac{d}{dx}[e^x] + \frac{d}{dx}[\sin x]$$

$$= -5e^x + \cos x \quad \checkmark$$

(5) $g(x) = \dfrac{10x + \sqrt{x}}{x^2}$ ; find $g'(x)$

$$g(x) = \frac{10x}{x^2} + \frac{\sqrt{x}}{x^2}$$

$$= \frac{10x}{x^2} + \frac{x^{1/2}}{x^2}$$

$$= 10x^{-1} + x^{-3/2}$$

$$g'(x) = \frac{d}{dx}[10x^{-1} + x^{-3/2}]$$

$$= 10\frac{d}{dx}[x^{-1}] + \frac{d}{dx}[x^{-3/2}]$$

$$= 10(-1)x^{-1-1} - 3/2\, x^{-3/2-1}$$

$$= -10x^{-2} - 3/2\, x^{-5/2}$$ ✓