Lecture 7: Instantaneous rate of change

HW 6 # 14 | find the value of x such that the derivative of $y = 26e^x$ is 1.

find x s.t. $y'(x) = 1$

$$y'(x) = \frac{d}{dx}\left[26e^x\right] = 26e^x$$

$$26e^x = 1$$
$$e^x = 1/26$$
$$\ln(e^x) = \ln(1/26)$$
$$x = \ln(1/26) = -\ln 26$$

Recall: the derivative of $f(x)$ measures the "slope" of $f(x)$

another way: $f'(c) = $ instantaneous $\overbrace{\text{rate of change}}^{\text{R.O.C.}}$ of f at $x = c$

e.g. ① Let $s(t) = 8t^2 + 3t - 1$ (meters) represent the position of an object moving on a line at time t (sec). Find the velocity of the object.

Physics: velocity = R.o.C of position
= derivative of position

velocity: $v(t) = \frac{d}{dt}\left[8t^2 + 3t - 1\right] = 16t + 3$

What is the velocity at $t = 10$.

$$v(10) = 16 \cdot 10 + 3 = 163 \text{ m/s}.$$

(2) $P(a) = \cos a + \frac{1}{3} a^3$

$P(a)$: represents profit in thousands of \$

a : represents money spent on ads in thousands of \$

derivative $P(a)$

What is the instantaneous r.o.c of profit with respect to money spent on ads?

a

$$\text{find} \quad P'(a) = \frac{d}{da}\left[\cos a + \frac{1}{3} a^3\right]$$

$$= -\sin a + \frac{1}{3} \cdot 3 \cdot a^2$$

$$= a^2 - \sin a$$

(3) The population of a city is

$$P(t) = 800t^2 - t + 100$$

where t is number of years since the year 2000

In which year is the population increasing at a rate of 3199 people/year.

↑ derivative

$P'(t) = 3199$ solve for t.

$$P'(t) = \frac{d}{dt}\left[800t^2 - t^1 + 100\right]$$

$$= 800 \cdot 2 \cdot t^1 - 1 \cdot 1 t^0 + 0$$

$$= 1600t - 1$$

$$1600t - 1 = 3199$$

$$1600t = 3200$$

$t = 2$. In the 2002 the r.o.c. of pop $= 3199$ people/year

(4) Suppose F = temp in °F
C = temp in °C

$$C = \frac{2}{3}(F - 10)$$

What is r.o.c of C wrt. F?

treat F as variable

$$C' = \frac{2}{3}(1 - 0) = 2/3$$

What is roc. of F wrt. C?

Solve for F treat C as a variable.

$$\frac{3}{2}C = F - 10$$

$$F = \frac{3}{2}C + 10$$

$$F' = \frac{3}{2}C^0 + 0 = \frac{3}{2}$$

(5) What is the $\overbrace{\text{rate of change}}^{\text{derivative}}$ of the area of a circle $\underbrace{\text{wrt. the radius}}_{\text{treat } r \text{ as the variable}}$ when $r = 6.5$.

$$A = \pi r^2$$

$$A' = \frac{d}{dr}[\pi r^2] = \pi \frac{d}{dr}[r^2] = 2\pi r$$

$$A'(6.5) = 2\pi \cdot 6.5 = 13\pi$$

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HW 6 #11 | find equation for tangent line to $y = 4\sin x$ at $x = \pi$.

$$y - y_1 = m(x - x_1) \qquad (x_1, y_1) = (\pi, y(\pi))$$

$$m = y'(\pi)$$

$$(x_1, y_1) = (\pi, 0)$$

$$y' = \frac{d}{dx}[4\sin x] = 4\frac{d}{dx}[\sin x] = 4\cos x$$

$$m = y'(\pi) = 4\cos(\pi) = -4$$

$$y - 0 = -4(x - \pi)$$

$$y = -4(x - \pi)$$ ← tan line at $x = \pi$ to y.

Recall: the derivative of $f(x)$ measures the "slope" or "steepness" of f at x.

another way: $f'(c) =$ instantaneous $\overbrace{\text{rate of change}}^{\text{r.o.c.}}$ of f at $x = c$

e.g. ① Let $s(t) = 8t^2 + 3t - 1$ (metres) represent the position of a an object moving along a straight line at time t (sec). Find the velocity.

Physics : velocity = rate of change of metres with respect to (wrt.) time in seconds

$$= \frac{d}{dt}\left[s(t)\right] = s'(t)$$

velocity : $v(t) = \frac{d}{dt}\left[8t^2 + 3t - 1\right]$
$= 8 \cdot 2t^1 + 3 \cdot 1t^0 + 0$
$= 16t + 3$

What is the velocity at $t = 10$ sec.
$v(10) = 16 \cdot 10 + 3 = 163$ m/s

② $P(a) = \cos a + \frac{1}{3}a^3$
$P(a)$: represents profit in thousands of \$
a : represents money spent on ads in thousands of \$

What is the $\overbrace{\text{instantaneous}}^{\text{derivative}}$ r.o.c. of $\overbrace{\text{profit}}^{P}$ wrt. $\underbrace{\text{money spent on ads}}_{a}$?

⇝ find $P'(a) = \left(\frac{d}{da}\left[P(a)\right]\right)$

$$P'(a) = \frac{d}{da}\left[\cos a + \frac{1}{3}a^3\right] = -\sin a + \frac{1}{3} \cdot 3 \cdot a^2$$
$$= a^2 - \sin a \quad ✓$$

③ The population of a city is
$P(t) = 800t^2 - t + 100$
where t is number of years since the year 2000.
In which year is the population increasing at a rate of 3199 people/year?

derivative

$P'(t) = 3199$ solve for t.

$$P'(t) = \frac{d}{dt}\left[800t^2 - t^1 + 100\right]$$

$$= 800 \cdot 2t^1 - 1 \cdot 1t^0 + 0$$

$$= 1600t - 1$$

$$1600t - 1 = 3199$$
$$1600t = 3200$$
$$t = 2$$

In the year 2002 pop. inc. at a rate of 3199 people/year.

(4) Suppose C = temp in °C
F = temp in °F

$C = 3F + 10.$

a) what is the $\underbrace{\text{r.o.c. of } C}_{\text{derivative of } C}$ $\underbrace{\text{wrt. } F}_{\text{treat } F \text{ as variable}}$?

$$C' = \frac{d}{dF}[C] = \frac{d}{dF}[3F + 10] = 3 + 0 = 3$$

b) what is the $\underbrace{\text{r.o.c of } F}_{\text{derivative } F}$, $\underbrace{\text{wrt } C}_{\text{treat } C \text{ as variable}}$?

Solve for F: $C - 10 = 3F$

$$F = \frac{1}{3}C - \frac{10}{3}$$

$$F' = \frac{d}{dC}[F] = \frac{d}{dC}\left[\frac{1}{3}C - \frac{10}{3}\right] = \frac{1}{3} + 0 = \frac{1}{3}$$

(5) What is the $\overbrace{\text{r.o.c. of the area}}^{\text{derivative of } A}$ of a circle $\underbrace{\text{wrt. its radius}}_{\text{treat } r \text{ as var.} \rightsquigarrow d/dr}$ when $r = 6.5$?

$$A = \pi r^2$$

$$A' = \frac{d}{dr}[\pi r^2] = \pi \frac{d}{dr}[r^2] = 2\pi r$$

$$A'(6.5) = 2\pi(6.5) = 13\pi. \checkmark$$