Lecture 8 : Product rule.

Product rule :

$$\frac{d}{dx}\left[f(x) \cdot g(x) \right] = \underbrace{f'(x)}_{\uparrow \text{deriv}} \cdot g(x) + f(x) \cdot \underbrace{g'(x)}_{\uparrow \text{deriv}}$$

very wrong : $\frac{d}{dx}\left[f(x)\,g(x)\right] \cancel{=} f'(x)\,g'(x)$ ← don't do this

e.g. ① $f(x) = \underbrace{(x^{16} + 3x + 5)}_{h(x)}\underbrace{(2x^{-5} + x + 3)}_{g(x)}$; find $f'(x)$

$$f(x) = h(x) \cdot g(x)$$

$$f'(x) = \frac{d}{dx}\left[(x^{16} + 3x + 5)(2x^{-5} + x + 3) \right]$$

$$= \underbrace{(16x^{15} + 3 + 0)}_{h'(x)}\underbrace{(2x^{-5} + x + 3)}_{g(x)} + \underbrace{(x^{16} + 3x + 5)}_{h(x)}\underbrace{(2\cdot(-5)x^{-6} + 1 + 0)}_{g'(x)}$$

$$= (16x^{15} + 3)(2x^{-5} + x + 3) + (x^{16} + 3x + 5)(-10x^{-6} + 1)$$

② $y = \underbrace{6\sqrt[4]{x^3}}_{h(x)}\ \underbrace{(1 + 3x)}_{g(x)}$; $y'(4) = ?$

$\sqrt[4]{x^3} = x^{3/4}$

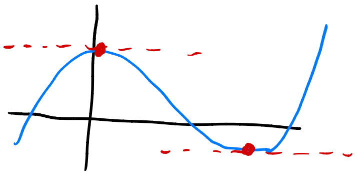
$$y = h(x)\,g(x) \qquad y' = \frac{d}{dx}\left[(6\sqrt[4]{x^3})(1 + 3x) \right]$$

$$= \left(6 \cdot \frac{3}{4} x^{3/4 - 1}\right)(1 + 3x) + 6\sqrt[4]{x^3}\,(0 + 3)$$

$$= \frac{9}{2} x^{-1/4}(1 + 3x) + 18\sqrt[4]{x^3}$$

$$y'(4) = \frac{9}{2}(4)^{-1/4}(1 + 3\cdot 4) + 18\sqrt[4]{4^3} = \frac{117}{2\sqrt{2}} + 36\sqrt{2}$$

③ find all x such that $y = 8x^6e^x$ has a horizontal tangent line.



when does y have 0 "steepness"
0 "slope"
0 derivative

$y'(x) = 0$ solve for x.

$$y' = \frac{d}{dx}\left[8x^6e^x\right] = 8\cdot 6x^5e^x + 8x^6e^x$$
$$= 8x^5e^x(6 + x)$$

$y' = 0$ ⇝ $8x^5e^x(6+x) = 0$

⇝ $8x^5e^x = 0$ → more complicated
or
$6 + x = 0$ → $x = -6$

$$\frac{8x^5e^x}{e^x} = \frac{0}{e^x}$$ $e^x \neq 0$

$8x^5 = 0$
$x = 0$

at $x = -6, 0$ y has a horizontal tangent line. ✓

Lecture 8: Product rule

Product rule:

$$\frac{d}{dx}\left[h(x)\cdot g(x)\right] = h'(x)g(x) + h(x)g'(x)$$

wrong way: $\frac{d}{dx}\left[h(x)g(x)\right] \neq h'(x)g'(x)$ Bad don't do this :(

e.g. ① $f(x) = (x^{16}+3x+5)(2x^{-5}+x+3)$; find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[(x^{16}+3x+5)(2x^{-5}+x+3)\right]$$

$$= (x^{16}+3x+5)'(2x^{-5}+x+3) + (x^{16}+3x+5)(2x^{-5}+x+3)'$$

$$= (16x^{15}+3+0)(2x^{-5}+x+3) + (x^{16}+3x+5)(2\cdot(-5)x^{-6}+1+0)$$

$$= (16x^{15}+3)(2x^{-5}+x+3) + (x^{16}+3x+5)(-10x^{-6}+1)$$

② $y = 6\sqrt[4]{x^3}\,(1+3x)$; $y'(4) = ?$

$$y' = \frac{d}{dx}\left[(6\sqrt[4]{x^3})(1+3x)\right]$$

Recall $6\sqrt[4]{x^3} = 6x^{3/4}$

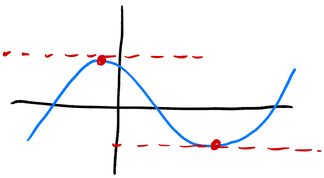
$$= (6\sqrt[4]{x^3})'(1+3x) + (6\sqrt[4]{x^3})(1+3x)'$$

$$= \left(6\cdot\frac{3}{4}x^{3/4-1}\right)(1+3x) + (6x^{3/4})(0+3)$$

$$= \frac{9}{2}x^{-1/4}(1+3x) + 18x^{3/4}$$

$$y'(4) = \frac{9}{2}(4)^{-1/4}(1+3(4)) + 18(4)^{3/4} = \frac{117}{2\sqrt{2}} + 36\sqrt{2}$$

(3) find all x such that $y = 8x^6 e^x$ has a horizontal tangent line.



when is the slope of tan line $= 0$

what values of x give $y'(x) = 0$.

$y'(x) = 0$ solve for x.

$$y' = \frac{d}{dx}\left[(8x^6)e^x\right] = (8x^6)'e^x + (8x^6)(e^x)'$$

$$= 8 \cdot 6x^5 e^x + 8x^6 e^x$$

$$= 8x^5 e^x (6 + x)$$

$y' = 0 \rightsquigarrow 8x^5 e^x (6 + x) = 0$

$\rightsquigarrow$ $8x^5 e^x = 0 \rightsquigarrow$ more complicated

or $6 + x = 0 \rightsquigarrow x = -6$

$$\frac{8x^5 e^x}{e^x} = \frac{0}{e^x}$$

Recall: $e^x \neq 0$ for any x

$$8x^5 = 0$$

$$x = 0$$

y has horizontal tan. lines at $x = -6, 0$.