MA 262 Section 596/597 Quiz 10

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April 24, 2023

Problem 1. Write your name, section and quiz number at the top of a full sized blank sheet of paper.

Problem 2. Find a particular solution y_p of the following equation

$$y'' - y' - 2y = 6x + 4.$$

Solution: If we consider the homogeneous part of the equation

$$y'' - y' - 2y = 0,$$

then we can calculate the characteristic equation $r^2 - r - 2 = 0$. The characteristic equation has roots $r = -1, 2$. Hence the homogeneous solution will be of the form

$$y_h = c_1 e^{-x} + c_2 e^{2x}$$

for real constants c_2 and c_2. Now let us examine the right hand side of the differential equation, $6x + 4$. The particular solution will be some linear combination of the derivatives of the terms in $6x + 4$. Hence y_p is a linear combination of x and 1. If A and B are real numbers, then $y_p = Ax + B$. Now we will need to find which values of A and B make y_p a solution. To acomplish this we will plug in y_p into the original differential equation. Since

$$y_p' = A \text{ and } y_p'' = 0,$$

then

$$0 - (A) - 2(Ax + B) = 6x + 4.$$

Combining like terms we see that

$$-2Ax + (-A - 2B) = 6x + 4.$$

Since x and 1 are linearly independent, then

$$-2A = 6 \text{ and } -A - 2B = 4.$$

Hence $A = -3$ and $B = -1/2$ and $y_p = -3A - 1/2$.

Problem 3. Find the eigenvalues of the following matrix

$$\begin{pmatrix} 0 & 2 \\ -32 & 0 \end{pmatrix}.$$

Solution: First we will find the characteristic equation of the matrix

$$\det(A - \lambda I) = \det \begin{pmatrix} -\lambda & 2 \\ -32 & -\lambda \end{pmatrix} = \lambda^2 + 64.$$

The eigenvalues are then the roots of the characteristic equation $\lambda = \pm 8i$.