MA 262 Section 596/597 Quiz 11

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Problem 1. Write your name, section and quiz number at the top of a blank full sized sheet of paper.

Problem 2. Apply the eigenvalue method to find a general solution of the given system:

$$x_1' = 3x_1 - 5x_2$$
$$x_2' = x_1 - 3x_2.$$

Solution: Let A be the coefficient matrix for the system of equations. If $x = (x_1, x_2)$, then the system of differential equations becomes the linear system $x' = Ax$. Hence

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

The first step in the eigenvalue methods is to compute the eigenvalues of the matrix A. Thus we must solve for lambda in the equation

$$0 = \det(A - \lambda I) = \det \begin{pmatrix} 3-\lambda & -5 \\ 1 & -3-\lambda \end{pmatrix} = \lambda^2 - 4.$$

Thus the eigenvalues for A are ± 2. The next step is to find the associated eigenvectors v for each eigenvalue. Starting with $\lambda = 2$, if v is an eigenvector for 2 then

$$(A - 2I)v = 0.$$

Hence we have to determine a basis for the nullspace of $A - 2I$. In order to proceed we will have to put the matrix $A - 2I$ into row-echelon form:

$$A - 2I = \begin{pmatrix} 1 & -5 \\ 1 & -5 \end{pmatrix} \to \begin{pmatrix} 1 & -5 \\ 0 & 0 \end{pmatrix}.$$

Hence the nullspace is spanned by the vector $(5, 1)$. We may take the eigenvector for $\lambda = 2$ to be $v_1 = (5, 1)$.

Now we repeat the process for $\lambda = -2$. In this case we want to find a basis for the nullspace of the matrix $A + 2I$. A similar computation as above yeilds the eigenvector $v_2 = (1, 1)$.

Finally, we have that

$$x(t) = c_1 v_1 e^{2t} + c_2 v_2 e^{-2t}.$$

Problem 3. Which topics or ideas in this course did you find the most enjoyable? Which topics did you find to be harder than the rest?