MA 262 Section 596/597 Quiz 2

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Problem 1. Write your name and section number on a full sheet of blank paper.

Problem 2. Find the general solution of the form $y(x)$ to the differential equation

$$\frac{dy}{dx} = 20\sqrt{xy}.$$

Assume x and y are positive.

Solution: This differential equation is seperable since we may write it in the form

$$\frac{dy}{\sqrt{y}} = 20\sqrt{x}dx.$$

Now if we integrate both sides of the above equation we obtain:

$$\begin{aligned} \int \frac{dy}{\sqrt{y}} &= \int 20\sqrt{x}dx \\ 2\sqrt{y} &= \frac{40}{3}x^{3/2} + c \\ y(x) &= (\frac{20}{3}x^{3/2} + c)^2. \end{aligned}$$

Problem 3. Find the general solution to the differential equation, then use the initial conditions to find the corresponding particular solution.

$$xy' - y = x, \quad y(1) = 21.$$

Assume $x > 0$.

Solution: If we multiply the differential equation by $1/x$ we will have

$$y' - \frac{1}{x}y = 1$$

which we can be solved using an integrating factor. Let $I(x) = e^{\int -1/x dx} = 1/x$, then

$$\frac{d}{dx}[I(x)y] = \frac{y'}{x} - \frac{y}{x^2} = I(x)(y' - \frac{1}{x}y) = I(x).$$

Now if we integrate

$$I(x)y = \int I(x)dx$$

$$y(x) = x\ln x + cx.$$

Since $y(1) = 21$, then

$$21 = y(1) = (1)\ln(1) + c(1) = c.$$

Hence the particular solution to the differential equation with given initial condition is

$$y(x) = x\ln x + 21x.$$