MA 262 Section 596/597: Quiz 3

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Problem 1. Write your name and section number on a full sized blank piece of paper.

Problem 2. Find the general solution to the differential equation

$$x^2y' = xy + 2y^2.$$

Express the general solution in the form $y(x) =$.

Solution: First divide the equation by x^2 to obtain

$$y' = \frac{xy + 2y^2}{x^2}.$$

If we multiply the numerator and denominator of the above equation by $1/x^2$, then

$$y' = \frac{y/x + 2(y/x)^2}{1}.$$

Now let $vx = y$ (equivalently $v = y/x$), then $y' = v + 2v^2$. If we differentiate our change of variables $vx = y$, then we obtain

$$v + x\frac{dv}{dx} = \frac{dy}{dx} = v + 2v^2.$$

Hence

$$x\frac{dv}{dx} = 2v^2$$

which is seperable. By seperating our variables and integrating we obtain

$$\begin{aligned} \int \frac{dv}{v^2} &= \int \frac{2}{x}dx \\ \frac{-1}{v} &= 2\ln|x| + c \\ \frac{-x}{y} &= 2\ln|x| + c \\ y &= \frac{-x}{2\ln|x| + c} \end{aligned}$$

Problem 3. Given the differential equation

$$(6x + 9y)dx + (9x + 6y)dy = 0.$$

(a) Verify that the equation is exact.

(b) Find the general solution in the form $F(x, y) = c$ for some constant c.

Solution:

(a) Since $d/dy[6x + 9y] = 9 = d/dx[9x + 6y]$, then the differential equation is exact.

(b) $F(x,y) = \int 6x + 9y dx = 3x^2 + 9xy + c(y)$ and

$$9x + 6y = \frac{d}{dy}[F(x,y)] = \frac{d}{dy}[3x^2 + 9xy + c(y)] = 9x + c'(y).$$

Thus $c'(y) = 6y$ and consequently, $c(y) = \int 6y dy = 3y^2 + c_1$ for some constant c_1. By consolidating all constants we see that

$$3x^2 + 9xy + 3y^2 = c$$

is the general solution to the differential equation.