MA 262: Quiz 4 Section 596/597

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Problem 1. Given the differential equation

$$\frac{dx}{dt} = x^2 - 1$$

(a) find all the critical points,

(b) graph the phase diagram,

(c) and classify the critical points as either stable, semistable or unstable.

Solution:

(a) Critical points occur when $dx/dt = 0$, i.e., when $x^2 - 1 = 0$. The values of x that satisfy this equation are $x = \pm 1$.

(b) In order to draw the phase diagram we will need to know the value of dx/dt in between the critical points. This leaves us with three cases to check. If $x < -1$, then $dx/dt > 0$; if $-1 < x < 1$, then $dx/dt < 0$; and if $x > 1$, then $dx/dt > 0$.

(c) The critical point $x = -1$ is stable since at $x = -1$ the derivative switches from positive to negative. The critical point $x = 1$ is unstable since the derivative switches from negative to positive.

Problem 2. Given the differential equation

$$\frac{dx}{dt} = -x^2$$

(a) find all the critical points,

(b) graph the phase diagram,

(c) and classify the critical points as either stable, semistable or unstable.

Solution:

(a) Critical points occur when $dx/dt = 0$, i.e., when $-x^2 = 0$. The values of x that satisfy this equation are only at $x = 0$.

(b) In order to draw the phase diagram we will need to know the value of dx/dt in between the critical points. In this case there are only two intervals to check, namely $x < 0$ and $x > 0$. First if $x < 0$, then $dx/dt < 0$ and if $x > 0$, then we again have that $dx/dt < 0$.

(c) Since the derivative does not switch signs at the critical point $x = 0$, then this point is unstable.