MA 262 Section 596/597 Quiz 5

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Problem 1. Write your name, quiz number, and section number on a blank sheet of paper.

Problem 2. The general solution to the differential equation

$$y'' + 4y = 0$$

is of the form $y(x) = A\cos 2x + B\sin 2x$. If $y(0) = 5$ and $y'(0) = 14$, then find the particular solution corresponding to these initial conditions.

Solution: First the derivative of $y(x)$ is given by

$$y'(x) = -2A\sin 2x + 2B\cos 2x.$$

Now if we use the initial conditions we see that

$$5 = y(0) = A\cos(0) + B\sin(0) = A$$

and

$$14 = y'(0) = -2A\sin(0) + 2B\cos(0) = 2B.$$

This leaves us with the system of equations

$$\begin{aligned} A &= 5 \\ 2B &= 14. \end{aligned}$$

This system has solution $A = 5$ and $B = 7$. Thus the particular soltuion is

$$y(x) = 5\sin 2x + 7\cos 2x.$$

Problem 3. Let

$$\begin{aligned} 6x + 3y &= 0 \\ 12x + ky &= 0. \end{aligned}$$

What values of k does the system have a unique solution?

Solution: If we take the first equation multiply it by -2 and add it to the second equation we will have the system

$$\begin{aligned} 6x + 3y &= 0 \\ (k-6)y &= 0. \end{aligned}$$

We have two cases depending on if k is 6 or not. First if $k = 6$, then we will be left with one equation $6x+3y = 0$. This equation has infinite solutions given by $x = t$ and $y = -2t$ for any real number t. If $k \neq 6$, then we may divide by $k - 6$ in our second equation ($k - 6 \neq 0$) and obtain the system

$$\begin{aligned} 6x + 3y &= 0 \\ y &= 0. \end{aligned}$$

This system has a unique solution $x = 0$ and $y = 0$. Therefore, we will obtain a unique solution if and only if $k \neq 6$.