MA 262 Section 596/597 Quiz 6

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Mar. 3, 2023

Problem 1. Write your name, quiz number, and section number at the top of a full sized blank sheet of paper.

Problem 2. Compute the determinant of the matrix

$$\begin{pmatrix} 9 & 0 & 0 & 5 \\ 5 & 7 & 3 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 5 \end{pmatrix}.$$

Solution: We will compute the determinant using expansion by cofactors:

$$\det \begin{pmatrix} 9 & 0 & 0 & 5 \\ 5 & 7 & 3 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 5 \end{pmatrix} = 2 \det \begin{pmatrix} 0 & 0 & 5 \\ 7 & 3 & -5 \\ 3 & 1 & 5 \end{pmatrix} = 2 \cdot 5 \det \begin{pmatrix} 7 & 3 \\ 3 & 1 \end{pmatrix} = 10(7-9) = -20.$$

Problem 3. Solve the linear system $au + bv + cw = 0$ to determine if the vectors u, v and w are linearly independent.

$$u = (2, 0, 3) \quad v = (7, 5, -3) \quad w = (4, -1, 1)$$

Solution: Recall that by definition the vectors u, v and w are linearly dependent if there exists real numbers a, b and c not all zero such that $au + bv + cw = 0$. In the language of matrices this means that

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = a \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + b \begin{pmatrix} 7 \\ 5 \\ -3 \end{pmatrix} + c \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2a + 7b + 4c \\ 5b - c \\ 3a - 3b + c \end{pmatrix} = \begin{pmatrix} 2 & 7 & 4 \\ 0 & 5 & -1 \\ 3 & -3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Now we see that u, v and w are linearly independent if and only if the only solution to the above matrix equation is the trivial solution, i.e., $a = b = c = 0$. There are multiple ways that one could approach the problem from here. Two options would be to compute the determinant of the coefficient matrix or to covert the matrix into row echelon form and preform back substitution. We will take the former option.

$$\det \begin{pmatrix} 2 & 7 & 4 \\ 0 & 5 & -1 \\ 3 & -3 & 1 \end{pmatrix} = 2 \det \begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix} + 3 \det \begin{pmatrix} 7 & 4 \\ 5 & -1 \end{pmatrix} = 2(5-3) + 3(-7-20) = -77$$

Since the determinant is not 0, then the only solution is when $a = b = c = 0$. Thus the vectors u, v and w are linearly independent.