MA 262 Section 596/597 Quiz 7

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Problem 1. Write your name, quiz number, and section number at the top of a blank full sized sheet of paper.

Problem 2. Which of the following form a basis for $\mathbb{R}^3$. Justify each answer.

(a)
$$\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\3\\0\end{pmatrix}, \begin{pmatrix}0\\0\\2\end{pmatrix}\}$$

(b)
$$\{\begin{pmatrix}7\\0\\2\end{pmatrix}, \begin{pmatrix}8\\-3\\1\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\}$$

(c)
$$\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\3\end{pmatrix}\}$$

Solution:

(a) Since
$$\det\begin{pmatrix}1&0&0\\0&3&0\\0&0&2\end{pmatrix} \neq 0$$
then the set is a basis.

(b) Any basis for a vector space will have the same number of elements. Since in (a) we saw that three vectors formed a basis for $\mathbb{R}^3$, then 4 vectors can not form a basis.

(c) Again 2 vectors can not form a basis for $\mathbb{R}^3$

Problem 3. Find a basis for the plane
$$x - 6y + 3z = 0$$
in $\mathbb{R}^3$.

Solution: Let $y = s$ and $z = t$, then $x = 6s - 3t$. Hence
$$\begin{array}{rcrcr} x &=& 6s &-& 3t \\ y &=& s & & \\ z &=& & & t \end{array}.$$
Therefore,
$$\begin{pmatrix}x\\y\\z\end{pmatrix} = s\begin{pmatrix}6\\1\\0\end{pmatrix} + t\begin{pmatrix}-3\\0\\1\end{pmatrix}.$$

Consequently the basis for the plane is

$$\{\begin{pmatrix}6\\1\\0\end{pmatrix}, \begin{pmatrix}-3\\0\\1\end{pmatrix}\}.$$

Problem 4. *Bonus* What is the definition of a basis?

Solution: A basis for a vector space is a subset of the vector space that spans the space and is linearly independent.