

MA 262 Section 596/597 Quiz 8

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Problem 1. Write your name, quiz number, and section number at the top of a blank full sized sheet of paper.

Problem 2. Find the general solution to the differential equation

$$y'' + 8y' - 9y = 0.$$

Solution: The characteristic equation for the differential equation is $(r + 9)(r - 1) = r^2 + 8r - 9 = 0$. Thus the general solution is $y(x) = c_1e^x + c_2e^{-9x}$.

Problem 3. Given $f(x) = 5x$, $g(x) = 10x^2$, and $h(x) = 8x - 8x^2$ then

(a) find real numbers a and b such that

$$8 \cdot f(x) + a \cdot g(x) + b \cdot h(x) = 0;$$

(b) and conclude that the functions f , g , and h are linearly dependent.

Solution:

(a) We have that $40x + 10ax^2 + 8bx - 8bx^2 = 0$. If we combine the x terms we obtain

$$(40 + 8b)x + (10a - 8b)x^2 = 0.$$

Since x and x^2 are linearly independent, then $40 + 8b = 0$ and $10a - 8b = 0$. Hence $b = -5$ and $a = -4$.

(b) Since we have expressed 0 as a nontrivial linear combination of f , g , and h then they are linearly dependent.

Problem 4. Bonus. State the definition of linear dependence.

Solution: A set of vectors A is linearly dependent if there exists vectors v_1, \dots, v_n in A and scalars c_1, \dots, c_n not all zero such that $c_1v_1 + \dots + c_nv_n = 0$.