MA 262 Section 596/597 Quiz 9

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Problem 1. Write your name, quiz number and section number at the top of a blank full sized sheet of paper.

Problem 2. Find the general solution to the differential equation

$$y'' - 12y' + 52y = 0.$$

Solution: The corresponding characteristic equation is $r^2 - 12r + 52 = 0$. This has roots

$$r = \frac{12 \pm \sqrt{144 - 208}}{2} = 6 \pm 4i.$$

Thus the general solution to the differential equation is

$$y(x) = c_1 e^{6x} \cos(4x) + c_2 e^{6x} \sin(4x).$$

Problem 3. A mass of 8 kg is attached to the end of a spring that is stretched .2 meters by a force of 40 N. It is set in motion with initial position $x_0 = 0$ meters and initial velocity $v_0 = 5$ meters per second. Find the function, $x(t)$, that describes the mass's position.

Solution: First we will need to find the spring constant. We may use the fact that

$$F_{pull} = k\Delta l$$

where F_{pull} is the force required to strech the spring Δl meters. Thus in our scenario, $F_{pull} = 40$ and $\Delta l = 0.2$ and hence $k = 200$.

Let $x(t)$ denote the position of the mass at time t, then Newton tells us that

$$F = ma = mx''$$

where F is the forces acting on the mass and m is the mass of the object. The only force acting on our object is the one due to the spring which is given by Hooke's law $-kx$. Thus

$$-kx = mx''$$

and consequently

$$mx'' + kx = 0.$$

If we plug in our values we get the differential equation

$$8x'' + 200x = 0.$$

The corresponding characteristic equation is $8r^2 + 200 = 0$ which has roots $r = \pm 5i$. Thus $x(t) = c_1 \cos(5t) + c_2 \sin(5t)$. In order to find c_1 and c_2 we will use our initial conditions given in the problem. Since at $t = 0$ we have that $x(0) = 0$ and $x'(0) = v(0) = 5$, then

$$0 = c_1 \cos(0) + c_2 \sin(0) = c_1$$

and

$$5 = -5c_1 \sin(0) + 5c_2 \cos(0) = 5c_2.$$

Then we see that $c_1 = 0$ and $c_2 = 1$. Thus

$$x(t) = \sin(5t).$$