Review of Vector Spaces

"Def": Is a set of vectors V where we can add vectors together and multiply them by real numbers (scalars).

Ex $\mathbb{R}^n$ n-dim Euclidean space

$$\mathbb{R}^n = \{ (a_1, \ldots, a_n) \text{ where } a_1, \ldots, a_n \text{ are real numbers} \}$$

$$n = 2, \quad \mathbb{R}^2 = \left\{ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \text{ where } a_1 \text{ and } a_2 \text{ are real numbers} \right\}$$

We define addition in the normal.

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

Also scalar multiplication

$$c \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} c a_1 \\ c a_2 \end{pmatrix}$$

<u>Ex</u> Polynomials

$$\mathbb{P} = \{ \text{of all polynomials, } f(x) = \overbrace{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0}^{\text{degree } = n} \text{ where } a_n, \ldots, a_0 \text{ are real \#} \}$$

$$\mathbb{P}(n) = \{ \text{all polynomials of degree at most } n \}$$

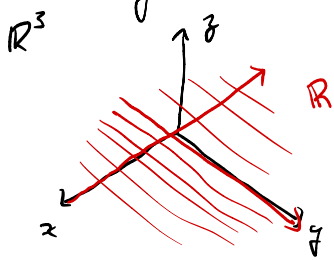
$$n = 2, \quad \mathbb{P}(2) = \{ f(x) = a_2 x^2 + a_1 x + a_0, \ a_2, a_1, a_0 \text{ are real \#} \}$$

Def. A subspace of a vector space V is a subset W of V that is also a vector space.

Ex Let $n \le m$ positive integers, then

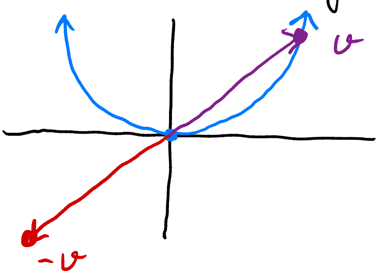
$\mathbb{R}^n$ is a subspace of $\mathbb{R}^m$

Say $n=2$ $m=3$, $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$



$\mathbb{R}^2$ is the plane $z=0$.

Ex $W = \{ (x,y)$ such that $y - x^2 = 0 \}$ in $\mathbb{R}^2$



Since $-v$ is not in W, then W is not a subspace of $\mathbb{R}^2$

Thm A subset W of a vector space is a subspace iff for all v and u in W and all real numbers c

$v + cu$ is in W.

Def. 1) Let $v_1, \dots, v_n$ be vectors in a vector space V, a linear combination of $v_1, \dots, v_n$ is a new vector

$$v = a_1 v_1 + a_2 v_2 + \dots + a_n v_n$$

where $a_1, \dots, a_n$ real #s.

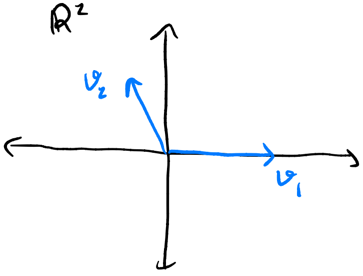
if $v = 0$, then the linear combination is called trivial.

2) $\operatorname{Span}(v_1, \dots, v_n) = \{\text{all linear comb. of } v_1, \dots, v_n\}$

this is a vector space, also the smallest vector space containing $v_1, \dots, v_n$

Ex) $\mathbb{R}_2$ $\quad v_1 = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad v_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$\mathbb{R}^2$



$\operatorname{Span}\begin{pmatrix} 2 \\ 0 \end{pmatrix} = x\text{-axis}$

$\operatorname{span}\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \text{line } y = -2x$

$\operatorname{span}\left\{\begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \end{pmatrix}\right\} = \mathbb{R}^2$

Review of Vector Spaces

"Def." A vector space is a set V with an operation of addition and an operation of scalar multiplication.

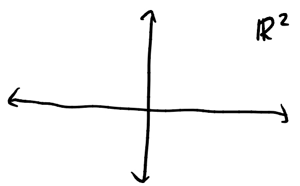
$\mathbb{R} = \{$ all real numbers $\}$ "number line"

Ex] $\mathbb{R}^n$ n-dimensional Euclidean space

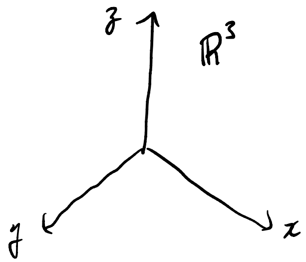
$\mathbb{R}^n = \{ (a_1, \dots, a_n)$ where $a_1, \dots, a_n$ are real #s$\}$

" (a_1, a_2)

$n = 2$, $\mathbb{R}^2 = \left\{ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \text{ where } a_1 \text{ and } a_2 \text{ real \# } \right\}$



$n = 3$, $\mathbb{R}^3 = \left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ where } a_1, a_2, a_3 \text{ real \# } \right\}$

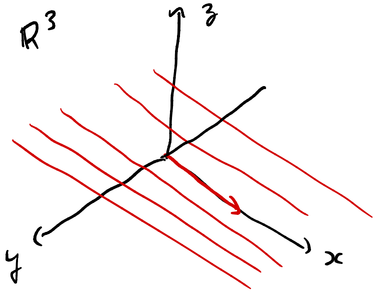


Ex] $\mathbb{P} = \{$ all polynomials, $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$
$a_n, \dots, a_0$ are real # $\}$

Def. A subset W of a vector space V is a subspace if W is also a vector space.

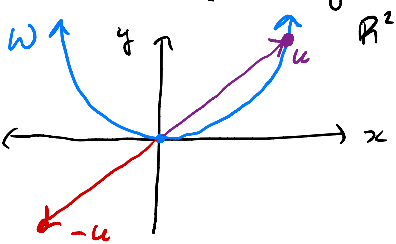
Ex) if $m < n$ are positive integers, then $\mathbb{R}^m$ is a subspace of $\mathbb{R}^n$

$m = 2, \quad n = 3 \quad , \quad \mathbb{R}^2$ in $\mathbb{R}^3$



$\mathbb{R}^2$ is the plane $z = 0$ in $\mathbb{R}^3$.

Ex) $W = \{ (x, y) \text{ such that } y - x^2 = 0 \}$ in $\mathbb{R}^2$



W is not a subspace since u is in W and $-u$ is <u>not</u> in W.

Let $v_1, \dots, v_n$ be vectors in a vector space V

Def. A linear combination of $v_1, \dots, v_n$ is a new vector of the form

$$v = a_1 v_1 + a_2 v_2 + \dots + a_n v_n$$

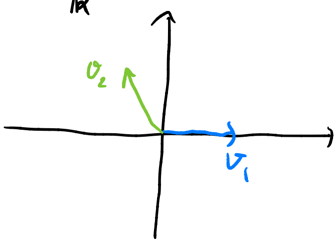
where $a_1, \dots, a_n$ are real #s.
if $a_1 = a_2 = \dots = a_n = 0$, then this is called a trivial linear comb.

Def. $\operatorname{span}\{v_1, \dots, v_n\} = \{$ all linear comb. of $v_1, \dots, v_n \}$

$\operatorname{span}\{v_1, \dots, v_n\}$ is the smallest subspace containing $v_1, \dots, v_n$.

Ex) $\mathbb{R}^2$ $\quad v_1 = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad v_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad v_3 = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$\mathbb{R}$



$\operatorname{span}\{v_1\} = x$-axis

$\operatorname{span}\{v_2\} = \; y = -2x$

$\operatorname{span}\{v_1, v_2\} = \mathbb{R}^2$