HW 28: #4 $\underbrace{y'' - 8y' + 4y}$ = $\underbrace{xe^x}$

homogeneous: $y'' - 8y' + 4y = 0$

$$r^2 - 8r + 4 = 0$$

$$r = \frac{8 \pm \sqrt{(-8)^2 - 4(4)}}{2}$$

$$= \frac{8 \pm \sqrt{48}}{2}$$

$$= \frac{8 \pm \sqrt{4 \cdot 4 \cdot 3}}{2}$$

$$= \frac{8 \pm 2 \cdot 2\sqrt{3}}{2}$$

$$= 4 \pm 2\sqrt{3}$$

homog. sol. $y_h = c_1 e^{(4+2\sqrt{3})x} + c_2 e^{(4-2\sqrt{3})x}$

Part. sol. We need to know all of the deriv. of xe^x

0th deriv. • $\boxed{xe^x}$

1st deriv • $\boxed{e^x} + xe^x$

2nd der • $e^x + e^x + xe^x$

$$y_p = Axe^x + Be^x$$

$$y_p' = Ae^x + Axe^x + Be^x$$

$$y_p'' = Ae^x + Ae^x + Axe^x + Be^x$$

$$= (2A + B)e^x + Axe^x$$

$$y'' - 8y' + 4y = xe^x$$

$$(2A+B)e^x + Axe^x - 8(Ae^x + Axe^x + Be^x) + 4(Axe^x + Be^x) = xe^x$$

$$(2A+B)e^x + Axe^x - 8Ae^x - 8Axe^x - 8Be^x + 4Axe^x + 4Be^x - xe^x = 0$$

$$(2A + B - 8A - 8B + 4B)e^x + (A - 8A + 4A - 1)xe^x = 0$$

$$-6A - 3B = 0$$

$$-3A = 1 \rightsquigarrow A = -1/3$$

$$-6\left(-\tfrac{1}{3}\right) - 3B = 0$$

$$3B = 2$$

$$B = 2/3$$

$$y_p = -\frac{1}{3}xe^x + \frac{2}{3}e^x$$

HW 33 #4|

$$x_1' = 2x_1 + x_2 + x_3$$
$$x_2' = -4x_1 - 3x_2 - x_3$$
$$x_3' = 4x_1 + 4x_2 + 2x_3$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad \frac{d\vec{x}}{dt} = \begin{pmatrix} 2 & 1 & 1 \\ -4 & -3 & -1 \\ 4 & 4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$0 = \det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 1 & 1 \\ -4 & 3-\lambda & -1 \\ 4 & 4 & 2-\lambda \end{vmatrix}$$

$$\begin{aligned} 0 &= (2-\lambda)\left((-3-\lambda)(2-\lambda)+4\right) \\ &\quad - 1\left(-4(2-\lambda)+4\right) \\ &\quad + 1\left(-16 - 4(-3-\lambda)\right) \\ &= (2-\lambda)(\lambda^2+\lambda-2) + 8 - 4\lambda - 4 - 16 + 12 + 4\lambda \\ &= (2-\lambda)(\lambda^2+\lambda-2) \\ &= -(\lambda-2)(\lambda+2)(\lambda-1) \end{aligned}$$

Eigenvalues $\lambda = 2, -2, 1$

Recall: $\quad Av = \lambda v \Rightarrow (A - \lambda I)v = 0$

$\lambda = -2$ | $\quad v_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$A - (-2)I = \left(\begin{array}{ccc|c} 4 & 1 & 1 & 0 \\ -4 & -1 & -1 & 0 \\ 4 & 4 & 4 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{aligned} a + b + c &= 0 \\ b + c &= 0 \\ c &= t \end{aligned} \quad \Rightarrow \quad \begin{aligned} a &= 0 \\ b &= -t \\ c &= t \end{aligned} \qquad \begin{pmatrix} a \\ b \\ c \end{pmatrix} = t\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \qquad v_1 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$(A - I)v = 0$$

$\underline{\lambda = 1}$ $\quad v_2 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$A - 1 \cdot I : \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -4 & -4 & -1 & 0 \\ 4 & 4 & 1 & 0 \end{array}\right)$$

$$\rightsquigarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = t \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \quad \rightsquigarrow \quad v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$\underline{\lambda = 2}$ $\quad v_3 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$A - 2I : \left(\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ -4 & -5 & -1 & 0 \\ 4 & 4 & 0 & 0 \end{array}\right) \rightsquigarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad v_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

then the general solution is

$$x = c_1 V_1 e^{-2t} + c_2 V_2 e^{t} + c_3 V_3 e^{2t}$$