HW 4 # 6 | A city has population of 24,000 in 1900 and a pop. of 31,000 in 1930. Assume pop. grows expotentially at a constant rate. What is pop. in 2000?

P = pop. of city

t = time in years after 1900

$P(0) = 24{,}000$

$P(30) = 31{,}000$

$$\frac{dP}{dt} = k P$$

$$dP = kP \cdot dt$$

$$\int \frac{dP}{P} = \int k \, dt$$

$$\ln|P| = kt + c$$

$$e^{\ln|P|} = e^{kt + c}$$

$$|P| = e^{c} \cdot e^{kt}$$

$$P = \boxed{\pm e^{c}} \cdot e^{kt} \qquad C = \pm e^{c}$$

$$P = C e^{kt}$$

$$24000 = P(0) = C e^{k \cdot 0} = C$$

$$31000 = P(30) = C e^{k \cdot 30}$$

$$31000 = 24000 e^{k \cdot 30}$$

$$\frac{31}{24} = e^{30k}$$

$$\ln \frac{31}{24} = 30k \qquad k = \frac{1}{30} \ln \frac{31}{24}$$

$$P(t) = 24000\, e^{\frac{1}{30} \ln\left(\frac{31}{24}\right) t}$$

$$P(100) = 2400\, e^{\frac{1}{30} \ln\left(\frac{31}{24}\right) \cdot 100} \approx 56{,}326 \text{ people.}$$

HW 3 # 5

$$\frac{dy}{dx} = 7x^6 y - y \quad ; \quad y(1) = -7$$

$$\frac{1}{y} \frac{dy}{dx} = 7x^6 - 1$$

$$\int \frac{dy}{y} = \int (7x^6 - 1)\, dx$$

$$\ln|y| = x^7 - x + c$$

$$e^{\ln|y|} = e^{x^7 - x + c}$$

$$|y| = e^c \cdot e^{x^7 - x}$$

$$y = \boxed{\pm e^c \cdot} e^{x^7 - x} \qquad C = \pm e^c$$

$$y = C e^{x^7 - x}$$

$$-7 = y(1) = C e^{(1)^7 - 1}$$

$$-7 = C e^0 = C$$

$$y = -7 e^{x^7 - x}$$

HW 4 #71 The ^{14}C in ancient bones was found to be 73% as much ^{14}C as current bones. The decay of ^{14}C is $k \approx 0.0001216$ for time in years.
How many years old are the ancient bones?

Radio active decay $\rightsquigarrow$ $\frac{dP}{dt} = -kP$

P = ammount of ^{14}C
t = time in years

$$\int \frac{dP}{P} = \int -k\, dt$$

$$\ln|P| = -kt + c$$

$$e^{\ln|P|} = e^{-kt+c}$$

$$|P| = e^{c} \cdot e^{-kt}$$
$$P = \boxed{\pm e^{c}} \cdot e^{-kt}$$
$$P(t) = Ce^{-kt}$$

$C = \pm e^{c}$

$$.73 = \frac{\text{amount of } ^{14}C \text{ in anchient bones}}{\text{" " " " current bones}}$$

$$= \frac{P(t_0)}{P(0)}$$ t_0 = age of anchient bones

$$= \frac{Ce^{-kt_0}}{Ce^{-k \cdot 0}} = \frac{e^{-kt_0}}{1} = e^{-kt_0}$$

$$.73 = e^{-0.0001216\, t_0}$$

$$\ln(.73) = \ln\left(e^{-0.0001216 t_0}\right)$$

$$\ln(.73) = -0.0001216 t_0$$

$$t_0 = \frac{\ln(.73)}{-0.0001216} \approx 2588 \text{ years old.}$$

<u>HW 4 #2</u> $(2+x)\frac{dy}{dx} = 2y$

$$\frac{(2+x)}{y}\frac{dy}{dx} = 2$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{2+x}$$

$$\int \frac{dy}{y} = \int \frac{2}{2+x}\,dx \qquad u = 2+x, \quad du = dx$$

$$\ln|y| = \int \frac{2}{u}\,du = 2\ln|u|$$

$$= 2\ln|2+x| + c$$

$$e^{\ln|y|} = e^{2\ln|2+x| + c}$$

$$|y| = e^c \cdot e^{\ln|2+x|^2}$$

$$|y| = e^c \cdot (2+x)^2$$

$$y = \boxed{\pm e^c} \cdot (2+x)^2 \qquad C = \pm e^c$$

$$y = C(2+x)^2$$

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