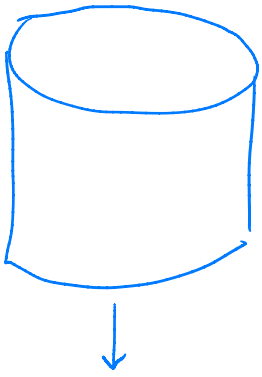
HW 6 # 2



$r_i = 2$ gal/min
$C_i = 3$ lb

$V(0) = 90$ gal
$x(0) = 0$ lb

$r_o = 3$ gal/min
$C_o = \frac{x}{V}$

tank will be empty after 90 min

a) Find the ammount of salt after t minutes

$$\frac{dx}{dt} = r_i C_i - r_o C_o$$

$$= 2 \cdot 3 - \frac{3x}{90-t}$$

$$V(t) = V(0) + t(r_i - r_o)$$

$$= 90 + t(2-3)$$

$$= 90 - t$$

$$\frac{dx}{dt} + \underbrace{\frac{3}{90-t}}_{P(t)} x = \underbrace{6}_{Q(t)}$$

$$I(t) = e^{\int P(t)\,dt} = e^{\int \frac{3}{90-t}\,dt} = e^{-3\ln(90-t)}$$

$$= \frac{1}{(90-t)^3}$$

$$\frac{d}{dt}\left[\frac{1}{(90-t)^3} x\right] = 6 \cdot \frac{1}{(90-t)^3}$$

$$\frac{x}{(90-t)^3} = \int \frac{6}{(90-t)^3}\,dt \qquad u = 90-t$$

$$\frac{x}{(90-t)^3} = \frac{3}{(90-t)^2} + C$$

$$x(t) = 3(90-t) + c(90-t)^3$$

$$0 = x(0) = 3(90-0) + c(90-0)^3$$

$$c = -3/90^2$$

$$x(t) = 3(90-t) - \frac{3}{90^2}(90-t)^3$$

HW 7 #7 $\quad 3xy^2y' = 33x^4 + y^3$

$$y' = 11x^3 y^{-2} + \frac{1}{3x} y$$

$$y' - \frac{1}{3x} y = 11x^3 y^{-2} \qquad \text{Bernulli } n=-2$$

$v = y^{1-n} = y^3 \qquad \frac{dv}{dx} = 3y^2 \frac{dy}{dx}$

$$y' = \frac{dy}{dx} = \frac{1}{3y^2} \frac{dv}{dx}$$

$$\frac{1}{3y^2} \frac{dv}{dx} - \frac{1}{3x} y = 11x^3 y^{-2}$$

$$\frac{dv}{dx} - \frac{1}{x} y^3 = 33x^3$$

$$\frac{dv}{dx} - \frac{1}{x} v = 33x^3 \qquad I(x) = e^{\int -1/x\, dx} = \frac{1}{x}$$

$$\frac{d}{dx}\left[\frac{1}{x} v\right] = 33x^2$$

$$\frac{1}{x} v = 11x^3 + c$$

$$v = 11x^4 + cx$$

$$y = \sqrt[3]{11x^4 + cx}$$

HW7 #21 $9x(x+4y)y' = 9y(x-4y)$

homogeneous equation since every term has the same degree

$$y' = \frac{9y(x-4y)}{9x(x+4y)} \qquad v = \frac{y}{x}$$

$$= \frac{y}{x} \cdot \frac{x-4y}{x+4y} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \qquad \frac{1}{x} = \frac{1}{x}$$

$$= \frac{\frac{y}{x}}{\frac{x}{x}} \cdot \frac{1-4\frac{y}{x}}{1+4\frac{y}{x}}$$

$$= \frac{v(1-4v)}{1+4v}$$

$xv = y \rightsquigarrow \frac{dy}{dx} = 1 \cdot v + x\frac{dv}{dx}$

$$\frac{v(1-4v)}{1+4v} = v + x\frac{dv}{dx}$$

$$x\frac{dv}{dx} = \frac{v(1-4v)}{1+4v} - v$$

$$= \frac{v - 4v^2 - v(1+4v)}{1+4v}$$

$$x\frac{dv}{dx} = \frac{-8v^2}{1+4v}$$

$$\int \frac{1+4v}{8v^2}\,dv = \int -\frac{1}{x}\,dx = -\ln|x| + c$$

$$\int \frac{1+4v}{8v^2}\,dv = \int \frac{1}{8v^2} + \frac{1}{2v}\,dv$$

$$= -\frac{1}{8v} + \frac{1}{2}\ln v$$

$$-\frac{1}{8v} + \frac{1}{2}\ln|v| = -\ln|x| + C$$

$$8 \times \left(\frac{-x}{8y} + \frac{1}{2}\ln|yx^{-1}| = -\ln|x| + C \right)$$

$$-xy^{-1} + 8\ln\sqrt{|yx^{-1}|} = -8\ln|x| + C$$

$$-xy^{-1} + 8\ln\sqrt{|yx^{-1}|} + 8\ln|x| = C$$

$$-xy^{-1} + 8\left(\ln\sqrt{|yx^{-1}|} + \ln|x|\right) = C$$

$$-xy^{-1} + 8\ln\sqrt{|yx^{-1}|\cdot|x|^2} = C$$

$$-xy^{-1} + 8\ln\sqrt{|xy|} = C$$

$$-xy^{-1} + 4\ln|xy| = C$$

HW7 #1

$(x + 4y)\,y' = 4x - y$

$$y' = \frac{4x - y}{x + 4y} \cdot \frac{1/x}{1/x}$$

$$= \frac{4 - y/x}{1 + 4\,y/x}$$

$$= \frac{4 - v}{1 + 4v}$$

$v = \frac{y}{x}$

$y = xv$

$y' = \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\frac{4 - v}{1 + 4v} = v + x\frac{dv}{dx}$$

$$x\frac{dv}{dx} = \frac{4 - v}{1 + 4v} - v = \frac{4 - v}{1 + 4v} - \frac{v(1 + 4v)}{1 + 4v} = \frac{4 - 2v - 4v^2}{1 + 4v}$$

$$\int \frac{1 + 4v}{4 - 2v - 4v^2}\,dv = \int \frac{1}{x}\,dx$$

$$-\frac{1}{2}\ln|4 - 2v - 4v^2| = \ln|x| + C$$

$$|4 - 2v - 4v^2|^{-1/2} = Cx$$

$$4 - 2v - 4v^2 = Cx^{-2}$$

$$4 - 2y/x - 4y^2/x^2 = Cx^{-2}$$

$$4x^2 - 2xy - 4y^2 = C$$

$$2x^2 - xy - 2y^2 = C$$