$$\frac{dy}{dx} = 20\sqrt{xy} \rightsquigarrow \int \frac{dy}{\sqrt{y}} = \int 20\sqrt{x}\,dx$$

$$2\sqrt{y} = \frac{40}{3}x^{3/2} + C$$

$$\left(\sqrt{y}\right)^2 = \left(\frac{20}{3}x^{3/2} + C\right)^2$$

$$\cancel{y = \frac{400}{9}x^3 + C}$$

$$(a+b)^2 \neq a^2 + b$$

$$xy' - y = x$$

$$y' \underbrace{\boxed{-\frac{1}{x}}}_{P(x)} y = 1$$

$$I(x) = e^{\int -1/x\,dx} = e^{-\ln x} = \frac{1}{x}$$

HW 9 #4 | Suppose the pop. of a country satisfies

$$\frac{dP}{dt} = k P(800 - P) \quad \text{with } k \text{ constant.}$$

- Pop in 1960 is 200 million and growing at a rate of 2 million per year.

Perdict the pop. in 2020.

$$\frac{dP}{dt} = kP(M - P)$$

P pop. in units of millions

t time in years after 1960

$$P_0 = P(0) = 200 \qquad \left.\frac{dP}{dt}\right|_{t=0} = 2 \qquad M = 800$$

$$P(t) = \frac{P_0 M}{P_0 + (M - P_0)e^{-kMt}}$$

$$2 = \left.\frac{dP}{dt}\right|_{t=0} = k\,P(0)\,(800 - P(0))$$

$$2 = k \cdot 200\,(800 - 200)$$

$$k = 1/60{,}000$$

$t = 60 \rightsquigarrow$ year 2020

$$P(60) = \frac{P_0 M}{P_0 + (M - P_0)e^{-kMt}} = \frac{200 \cdot 800}{200 + (800 - 200)e^{-\left(\frac{1}{60000}\right)800 \cdot 60}}$$

$$\approx 340.7 \text{ million.}$$

$$\frac{dy}{dx} = 20\sqrt{xy} \rightsquigarrow \int \frac{dy}{\sqrt{y}} = \int 20\sqrt{x}\,dx$$

$$2\sqrt{y} = \frac{40}{3} x^{3/2} + c$$

$$(\sqrt{y})^2 = \left(\frac{20}{3} x^{3/2} + c\right)^2$$

$$\cancel{y = \frac{400}{9} x^3 + c}$$

$$(a+b)^2 \neq a^2 + b^2$$

$$xy' - y = x$$

$$y' \boxed{-\frac{1}{x}} y = 1 \qquad I(x) = e^{\int -1/x\,dx} = e^{-\ln x} = \frac{1}{x}$$

$P(x)$

HW 9 # 41 Suppose the pop. of a country satisfies
$$\frac{dP}{dt} = k P(800 - P)$$ with k constant.

- Pop in 1960 was 200 million and growing at a rate of 2 million per year.

Predict the pop. in the year 2020.

$$\frac{dP}{dt} = kP(M - P)$$

P : pop in millions
t : time in years after 1960.

$P_0 = P(0) = 200 \qquad \left.\frac{dP}{dt}\right|_{t=0} \overset{?}{=} 2 \qquad M = 800$

$$P(t) = \frac{P_0 M}{P_0 + (M - P_0) e^{-kMt}}$$

$$\frac{dP}{dt} = k P(t) \cdot (800 - P(t))$$

$$2 = \left.\frac{dP}{dt}\right|_{t=0} = k \cdot P(0) \cdot (800 - P(0))$$

$$2 = k \cdot 200 \cdot (800 - 200)$$

$$k = 1/60{,}000$$

$t = 60 \rightsquigarrow$ year 2020

$$P(60) = \frac{P_0 M}{P_0 + (M - P_0) e^{-kMt}} = \frac{200 \cdot 800}{200 + (800 - 200) e^{-\frac{1}{60{,}000} \cdot 800 \cdot 60}}$$

≈ 340.7 million people.

HW 10 #2 $\frac{dx}{dt} = (x-1)^2$

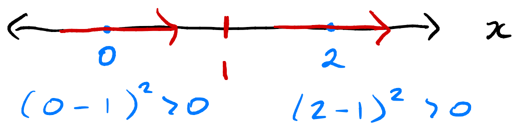
Finding / classifying the critical points.

Critical pts $\Rightarrow$ $dx/dt = 0$

$$\Rightarrow (x-1)^2 = 0$$
$$x = 1$$

Is $x=1$ stable, unstable, or semistable?

phase diagram



Semistable

$$\frac{dx}{dt} = (x-1)^2 \qquad x(0) = x_0$$

$$\int \frac{dx}{(x-1)^2} = \int dt$$

$$\frac{-1}{x-1} = t + c$$

$$\frac{-1}{x(0)-1} = 0 + c$$

$$\frac{1}{1-x_0} = c$$

$\Rightarrow$ Solve for $x(t)$

$$-1 = (x-1)(t+c)$$
$$-1 = xt + xc - t - c$$
$$x(t+c) = t + c - 1$$
$$x(t) = \frac{t+c-1}{t+c}$$
$$= \frac{t + 1/(1-x_0) - 1}{t + 1/(1-x_0)}$$
$$= \frac{(1-x_0)t + x_0 - 1}{(1-x_0)t + 1}$$