HW 13 #6

$7x + 4y = 0$
$14x + ky = 0$

What values of k does the equation
(a) have a unique solution
(b) no solution
(c) infinite solutions?

$$\begin{array}{c} R_1 \\ R_2 \end{array}\left(\begin{array}{cc|c} 7 & 4 & 0 \\ 14 & k & 0 \end{array}\right) \xrightarrow{-2R_1 + R_2} \left(\begin{array}{cc|c} 7 & 4 & 0 \\ 0 & k-8 & 0 \end{array}\right)$$

$$7x + 4y = 0$$
$$(k-8)y = 0$$

if $k \neq 8$, so $k - 8 \neq 0$, so $y = 0$

$$7x + 4(0) = 0$$
$$7x = 0$$
$$x = 0$$

we have a unique solution $x = 0$, $y = 0$

if $k = 8$, $(k-8)y = 0$

$$\rightsquigarrow (8-8)y = 0$$
$$0 \cdot y = 0$$
$$0 = 0$$

$$7x + 4y = 0$$
$$(k-8)y = 0 \leftarrow$$ this is useless

$\rightsquigarrow$

$$7x + 4y = 0$$

$y = t$

$$7x + 4t = 0$$
$$7x = -4t$$
$$x = -\frac{4}{7}t$$

we have an infinite amount of solutions

$x = -4/7\, t \quad y = t$

HW 12 #6 $y'' + 4y = 0$ solutions are of the form

$$y(x) = A\cos(2x) + B\sin(2x)$$

$y(0) = 5$, $y'(0) = 14$. Find A, B.

$$y'(x) = -2A\sin(2x) + 2B\cos(2x)$$

$$A(1) + B(0) = y(0) = 5$$

$$-2A(0) + 2B(1) = y'(0) = 14$$

$$A = 5$$
$$2B = 14$$

$A = 5$ $B = 7$

HW 13 51

$$\begin{aligned} 4x_1 - 5x_2 + 5x_3 &= 14 \\ 8x_1 - 10x_2 + 10x_3 &= 25 \\ -12x_1 + 15x_2 - 15x_3 &= -40 \end{aligned}$$

Solve for x_1, x_2, x_3.

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \left(\begin{array}{ccc|c} 4 & -5 & 5 & 14 \\ 8 & -10 & 10 & 25 \\ -12 & 15 & -15 & -40 \end{array}\right) \xrightarrow[\frac{1}{3}R_3 \to R_3]{\frac{1}{2}R_2 \to R_2} \left(\begin{array}{ccc|c} 4 & -5 & 5 & 14 \\ 4 & -5 & 5 & 25/2 \\ -4 & 5 & -5 & -40/3 \end{array}\right)$$

$$\xrightarrow{-R_1 + R_2 \to R_2} \left(\begin{array}{ccc|c} 4 & -5 & 5 & 14 \\ 0 & 0 & 0 & -3/2 \\ -4 & 5 & -5 & -40/3 \end{array}\right) \xrightarrow{R_1 + R_3 \to R_3} \left(\begin{array}{ccc|c} 4 & -5 & 5 & 14 \\ 0 & 0 & 0 & -3/2 \\ 0 & 0 & 0 & 2/3 \end{array}\right)$$

row-echlon

$$\begin{cases} 4x_1 - 5x_2 + 5x_3 = 14 \\ 0 = -3/2 \\ 0 = 2/3 \end{cases} \leftarrow$$

Bad nonsense!

So no solutions

$$\left(\begin{array}{ccc|c} 1 & -5/4 & 5/4 & 7/2 \\ 0 & 0 & 0 & -3/2 \\ 0 & 0 & 0 & 2/3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -5/4 & 5/4 & 7/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -5/4 & 5/4 & 7/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Quiz : $x^2 y' = xy + 2y^2$

$v = y/x$

$\downarrow$

$y = xv$

$d/dx \downarrow$ product rule

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$y' = \frac{y}{x} + 2\left(\frac{y}{x}\right)^2$$

$$\frac{dy}{dx} = y' = v + 2v^2$$

$$v + 2v^2 = v + x\frac{dv}{dx}$$

$$x\frac{dv}{dx} = 2v^2$$

$$\int \frac{dv}{v^2} = \int \frac{2}{x}\,dx$$

$$-\frac{1}{v} = 2\ln|x| + c$$

$$-\frac{x}{y} = 2\ln|x| + c$$

$$y = \frac{-x}{2\ln|x| + c}$$