HW 15 #8

$$x_1 \quad - x_3 + 7x_4 + 8x_5 = 0$$
$$x_2 + 6x_3 - 9x_4 + 4x_5 = 0$$

Write the system of equations as a matrix equation $Ax = 0$. Solve for x.

$$A = \begin{pmatrix} 1 & 0 & -1 & 7 & 8 \\ 0 & 1 & 6 & -9 & 4 \end{pmatrix} \qquad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$$

$$Ax = 0$$

A is in reduced-row-echelon form, so we may start back-substitutions.

$$\left.\begin{aligned} x_5 &= t \\ x_4 &= s \\ x_3 &= r \end{aligned}\right\} \text{free variables}$$

$$x_2 + 6x_3 - 9x_4 + 4x_5 = 0$$
$$x_2 + 6r - 9s + 4t = 0$$
$$x_2 = -6r + 9s - 4t$$

$$x_1 - x_3 + 7x_4 + 8x_5 = 0$$
$$x_1 - r + 7s + 8t = 0$$
$$x_1 = r - 7s - 8t$$

$$\begin{aligned} x_1 &= r - 7s - 8t \\ x_2 &= -6r + 9s - 4t \\ x_3 &= r \\ x_4 &= s \\ x_5 &= t \end{aligned}$$

$$\rightsquigarrow \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = r\begin{pmatrix} 1 \\ -6 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} -7 \\ 9 \\ 0 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -8 \\ -4 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Review Homogenous diff Eq.

$$xy' - x^2 y = xy^2$$

$$y' - \underbrace{xy}_{\text{deg}=2} = \underbrace{y^2}_{\text{deg}=2} \Longrightarrow \text{homo}$$

$$v = \frac{y}{x} \rightsquigarrow \quad y = xv$$

$$y' = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} - x(xv) = (xv)^2$$

$$v + x\frac{dv}{dx} = x^2 v + x^2 v^2$$

Spring 22 # 2 Which of the following are correct?

I. If we solve the initial value prob $y' = y^2$ $y(0) = \frac{1}{2}$ we conclude that its solution $y(x)$ is continuous on interval $(-1, 1)$, but not interval $(-3, 3)$.

$$y' = y^2 \; ; \quad y(0) = \frac{1}{2}$$

$$\int \frac{dy}{y^2} = \int dx$$

$$-\frac{1}{y} = x + c$$

$$y = \frac{-1}{x+c}$$

$$\frac{1}{2} = y(0) = \frac{-1}{0+c} = \frac{-1}{c} \rightsquigarrow c = -2$$

$$y = \frac{1}{2-x}$$

(1) Is y cont. on $(-1, 1)$? Yes!

(2) Does y have a discont. on $(-3, 3)$? Yes! at $x = 2$.

II. The functions $y(x) = mx$ solve the initial value prob $xy' = y$; $y(0) = 0$, for any m. So this initial value prob does not have a unique solution.

$$y = mx \rightsquigarrow y' = m$$

$$xy' = y$$

$$x(m) = mx \quad \checkmark$$

$$y(0) \stackrel{?}{=} 0$$

$$m(0) = 0$$

III. The initial value prob $y' = xy$, $y(0) = 1$ has a unique solution $y(x)$ which is continuous for all x.

$$y' = xy$$

$$\int \frac{dy}{y} = \int x \, dx$$

$$\ln y = \frac{1}{2} x^2 + c$$

$$\ln(y(0)) = \frac{1}{2}(0)^2 + c$$

$$0 = \ln(1) = C$$

$$\ln y = \frac{1}{2} x^2$$

$$y = e^{\frac{1}{2}x^2}$$

this is unique and cont. for all x