HW 21 #6

$$\begin{aligned} x_1 - 3x_2 - 9x_3 - 5x_4 &= 0 \\ 2x_1 + x_2 - 4x_3 + 11x_4 &= 0 \\ x_1 + x_2 - x_3 + 7x_4 &= 0 \end{aligned}$$

Find a basis for the solution space.

$$\begin{pmatrix} 1 & -3 & -9 & -5 \\ 2 & 1 & -4 & 11 \\ 1 & 1 & -1 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightsquigarrow A\vec{x} = \underline{0}$$

$$\left(\begin{array}{cccc|c} 1 & -3 & -9 & -5 & 0 \\ 2 & 1 & -4 & 11 & 0 \\ 1 & 1 & -1 & 7 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 0 & -4 & -8 & -12 & 0 \\ 0 & -1 & -2 & -3 & 0 \\ 1 & 1 & -1 & 7 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & -1 & 7 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & -12 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 1 & -1 & 7 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{aligned} x_1 + x_2 - x_3 + 7x_4 &= 0 \\ x_2 + 2x_3 + 3x_4 &= 0 \end{aligned}$$

$$\begin{aligned} x_2 &= -2x_3 - 3x_4 \\ x_1 &= -x_2 + x_3 - 7x_4 \\ &= 2x_3 + 3x_4 + x_3 - 7x_4 \\ &= 3x_3 - 4x_4 \end{aligned}$$

$$\left.\begin{aligned} x_3 &= s \\ x_4 &= t \end{aligned}\right\} \text{free variables}$$

$$
\begin{aligned}
x_1 &= 3s - 4t \\
x_2 &= -2s - 3t \\
x_3 &= s \\
x_4 &= t
\end{aligned}
\rightsquigarrow
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = s\begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ -3 \\ 0 \\ 1 \end{pmatrix}
$$

The basis for the solution space is

$$
\left\{ \begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -4 \\ -3 \\ 0 \\ 1 \end{pmatrix} \right\}
$$

Aside:
Basis = 1) lin ind
2) span ✓

HW 21 # 4 Subspace of $\mathbb{R}^4$ is the set of vectors (a, b, c, d) such that

$$a = 2c$$
$$b = 3d$$

Find a basis for the space.

$$\begin{aligned} a &= 2c \\ b &= 3d \\ c &= s \\ d &= t \end{aligned} \Big\} \text{ free} \quad \rightsquigarrow \quad \begin{aligned} a &= 2s + 0t \\ b &= 0s + 3t \\ c &= s + 0t \\ d &= 0s + t \end{aligned}$$

$$\rightsquigarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = s \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 3 \\ 0 \\ 1 \end{pmatrix}$$

basis $\left\{ \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 0 \\ 1 \end{pmatrix} \right\}$

Aside:

Basis = 1) lin. ind.
2) Span

$\mathbb{R}^2$ $\quad \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\}$ ✓ Spans $\mathbb{R}^2$
but
lin. dep.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 0 \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

HW 19 # 3

W is a subset of $\mathbb{R}^4$ of vectors $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$

such that $x_1 + 4x_2 + 7x_3 + 8x_4 = 0$

Is W a subspace of $\mathbb{R}^4$?

Def. Given a $\mathbb{R}$- vector space V a subset W of V is a subspace if W is also a $\mathbb{R}$- vector space

Thm. A subset W of a $\mathbb{R}$- vector space V is a subspace if

closed under add. → 1) if u and v in W, then $u+v$ in W

closed under scalar mult → 2) if c is a real number and u in W then $c \cdot u$ is in W

"closed under linear combination"

Let $v = (x_1, x_2, x_3, x_4)$ $u = (y_1, y_2, y_3, y_4)$ be in W. Thus $x_1 + 4x_2 + 7x_3 + 8x_4 = 0$ and

$$y_1 + 4y_2 + 7y_3 + 8y_4 = 0.$$

$v + u = (x_1 + y_1, x_2 + y_2, x_3 + y_3, x_4 + y_4)$

Is $v + u$ in W?

$$
\begin{aligned}
&1(x_1 + y_1) + 4(x_2 + y_2) + 7(x_3 + y_3) + 8(x_4 + y_4) \\
&= (x_1 + 4x_2 + 7x_3 + 8x_4) + (y_1 + 4y_2 + 7y_3 + 8y_4) \\
&= 0 + 0 \\
&= 0
\end{aligned}
$$

So $u + v$ is in W $\checkmark$

Let c be a real number. Is $c \cdot v = (cx_1, cx_2, cx_3, cx_4)$ in W?

$$
\begin{aligned}
&(cx_1) + 4(cx_2) + 7(cx_3) + 8(cx_4) \\
&= c(x_1 + 4x_2 + 7x_3 + 8x_4) \\
&= c \cdot 0 \\
&= 0
\end{aligned}
$$

So $c \cdot v$ is in W.

Thus by thm W is a subspace of V.