HW26 #7 $a x^3 y^{(3)} + b x^2 y'' + c x y' + k y = 0$

$a, b, c, k$ are constants and $x > 0$.

Apply sub $v = \ln x$

$$\leadsto \quad a \frac{d^3 y}{dv^3} + (b - 3a) \frac{d^2 y}{dv^2} + (c - b + 2a) \frac{dy}{dv} + ky = 0$$

Find general solution to $x^3 y^{(3)} + 10 x^2 y'' + 8 x y' = 0 \quad x > 0.$

$a = 1 \quad b = 10 \quad c = 8 \quad k = 0$

$$\frac{d^3 y}{dv^3} + 7 \frac{d^2 y}{dv^2} = 0$$

$$r^3 + 7 r^2 = 0$$
$$r^2 (r + 7) = 0$$
$$r = 0, 0, -7$$

$$y(v) = c_1 e^{0 \cdot v} + c_2 v e^{0 \cdot v} + c_3 e^{-7v}$$
$$= c_1 + c_2 v + c_3 e^{-7v}$$

$$y(x) = c_1 + c_2 \ln x + c_3 e^{-7 \ln x}$$
$$= c_1 + c_2 \ln x + c_3 x^{-7}.$$

HW 25 #81 $\quad 3y^{(3)} + 10y'' - 9y' - 4y = 0$

char. eq $\rightarrow \quad 3r^3 + 10r^2 - 9r - 4 = 0$

any solution $r = \frac{a}{b}$ w/ $a, b$ integers.
$a$ divides $4$ (last coef) and $b$ divides $3$ (first coef)

check $r = 1$. $\quad 3(1)^3 + 10(1)^2 - 9(1) - 4$

$$3 + 10 - 9 - 4$$

$$13 - 13$$

$$0 \quad \checkmark$$

$$3r^3 + 10r^2 - 9r - 4 = (r-1)(\quad\quad)$$

$$
\begin{array}{r|l}
 & 3r^2 + 13r + 4 \\
\hline
r-1 & 3r^3 + 10r^2 - 9r - 4 \\
 & -(3r^3 - 3r^2) \\
\hline
 & 13r^2 - 9r - 4 \\
 & -(13r^2 - 13r) \\
\hline
 & 4r - 4 \\
 & -(4r - 4) \\
\hline
 & 0
\end{array}
$$

$$3r^3 + 10r^2 - 9r - 4 = (r-1)(3r^2 + 13r + 4)$$
$$= (r-1)(3r+1)(r+4)$$

$$(r-1)(3r+1)(r+4) = 0$$
$$r = 1, \ -1/3, \ -4$$

$$y(x) = c_1 e^{1 \cdot x} + c_2 e^{-\frac{1}{3}x} + c_3 e^{-4x} \quad \checkmark$$

HW 26 #71 $\quad a x^3 y''' + b x^2 y'' + c x y' + k y = 0$

where $a, b, c, k$ are constants and $x > 0$

$$v = \ln x \rightsquigarrow a \frac{d^3 y}{dv^3} + (b - 3a)\frac{d^2 y}{dv^2} + (c - b + 2a)\frac{dy}{dv} + k y = 0$$

Find the general sol. to $x^3 y''' + 10 x^2 y'' + 8 x y' = 0 \quad x > 0.$

$a = 1 \quad b = 10 \quad c = 8 \quad k = 0$

$$\frac{d^3 y}{dv^3} + 7 \frac{d^2 y}{dv^2} = 0$$

$$r^3 + 7 r^2 = 0 \longleftarrow \text{char. eq.}$$

$$r^2 (r + 7) = 0$$

$$r = 0, 0, -7 \rightsquigarrow y_1(v) = c_1 e^{0 \cdot v} + c_2 v e^{0 \cdot v} + c_3 e^{-7v}$$
$$= c_1 + c_2 v + c_3 e^{-7v}$$

$v = \ln x$

$$y_2(x) = y_1(\ln x) = c_1 + c_2 \ln x + c_3 e^{-7 \ln x}$$
$$= c_1 + c_2 \ln x + c_3 x^{-7}.$$

HW 27 #6 $10y^{(4)} + 7y^{(3)} + 41y'' + 28y' + 4y = 0$

$y = \cos 2x$ is a solution to the differential eq.

Find the general solution.

char $\longrightarrow$ $10r^4 + 7r^3 + 41r^2 + 28r + 4 = 0$

$y = \cos 2x$ is a a solution to to the dif. eq

$\Rightarrow$ $r = \pm 2i$ is a solution to the char. eq.

Recall: if $r = a \pm bi$ is a solution to the char. eq, then

$c_1 e^{ax}\cos(bx) + c_2 e^{ax}\sin(bx)$

are terms in the general solution.

$$y = \cos(2x) = e^{0 \cdot x}\cos(2x)$$

$$r = 0 \pm 2i$$

So this tells us that both $r - 2i$ and $r + 2i$ divide the char. eq. ie $r^2 + 4 = (r-2i)(r+2i)$ divides the char. eq.

$$10r^4 + 7r^3 + 41r^2 + 28r + 4 = (r^2+4)(\quad)$$

$$
\begin{array}{r|l}
 & 10r^2 + 7r + 1 \\ \hline
r^2+4 & 10r^4 + 7r^3 + 41r^2 + 28r + 4 \\
 & -(10r^4 + 40r^2) \\ \hline
 & 7r^3 + r^2 + 28r + 4 \\
 & -(7r^3 + 28r) \\ \hline
 & r^2 + 4 \\
 & -(r^2 + 4) \\ \hline
 & 0
\end{array}
$$

$$10r^4 + 7r^3 + 41r^2 + 28r + 4 = (r^2+4)(10r^2+7r+1)$$
$$= (r^2+4)(2r+1)(5r+1)$$

$$(r^2+4)(2r+1)(5r+1) = 0$$
$$r = \pm 2i, \ -1/2, \ -1/5$$

$$y(x) = c_1 e^{0\cdot x}\cos 2x + c_2 e^{0\cdot x}\sin 2x + c_3 e^{-1/2 x} + c_4 e^{-\frac{1}{5}x}$$
$$= c_1 \cos 2x + c_2 \sin 2x + c_3 e^{-1/2 x} + c_4 e^{-1/5 x}$$