Lesson 25: Divergence Thm

Recall: If $F = \langle f, g \rangle$, then we defined the divergence of F as

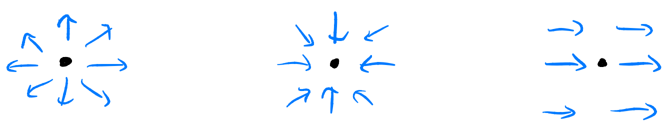
$$\operatorname{div} F = \nabla \cdot F = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}$$

Greens Thm: let R be a "good" region in the xy-plane with boundary C, then

$$\int_C \vec{F} \cdot \vec{n}\, ds = \iint_R \operatorname{div} F \, dA$$

flux along the bdy = accumulation of div. in the interior.

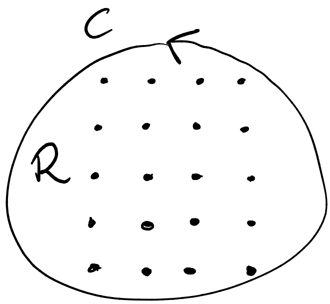
Earlier we saw $\operatorname{div} F$ measures how much fluid F is generating at any given pt.



div > 0 Source

div < 0 Sink

div = 0



F = describes water current
R = fountain with boundary C
each dot is either a water jet or a drain.

How much net water the fountain produces?

1) $\iint_R \operatorname{div} F \, dA$: walk in the interior of fountain, record how much water is being generated and drained.

2) $\int_C \vec{F} \cdot \vec{n} \, ds$: Walk around the fountain recording how much water flows through C.

Now in 3D, the same argument gives...

<u>Divergence Thm</u> E a "good" solid with boundary surface S,

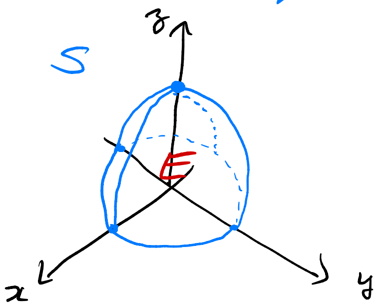
$$\iiint_E \operatorname{div} F \, dV = \iint_S \vec{F} \cdot \vec{n} \, dS \quad \left(= \iint_S \vec{F} \cdot d\vec{S} \right)$$

where $F = \langle f, g, h \rangle$, then

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z} = f_x + g_y + h_z$$

eg. Let $F = \langle x, y, z \rangle$, S be the surface bounded by $z = 0$ and above by $x^2 + y^2 + z^2 = 9$. Let $\vec{n}$ point outwards.

Compute the flux of F along S.



volume of sphere

$V = \frac{4}{3}\pi r^3$

$$\iint_S \vec{F} \cdot \vec{n} \, dS \overset{D.T.}{=} \iiint_E \operatorname{div} F \, dV$$

$$= \iiint_E \underbrace{\frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)}_{3} \, dV$$

$$= 3 \iiint_E dV$$

$$= 3 \cdot \frac{1}{2} \, \frac{4}{3} \pi (3)^3$$

$$= 54\pi$$

eg. Let $F = \langle -8y + x, -4x - 3y, 9y - 2z \rangle$, S is the cube $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, $-1 \leq z \leq 1$, let $\vec{n}$ point outwards. Find the flux along S.

$$\iint_S \vec{F} \cdot \vec{n} \, dS = \iiint \operatorname{div} F \, dV$$

$$= \iiint 1 + (-3) + (-2) \, dV$$

$$= -4 \iiint dV$$

$$= -4 (2)(2)(2)$$

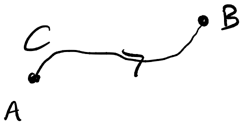
$$= -32$$

Overview:

Interval:

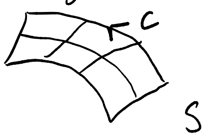
FTC: $\int_a^b \frac{d}{dt}[f]\,dt = f(b) - f(a)$

Curves:



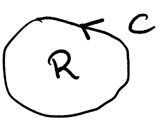
FTLI: $\int_C \nabla \varphi \cdot d\vec{r} = \varphi(B) - \varphi(A)$

Surfaces:



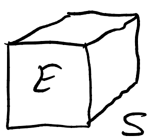
Stokes Thm: $\iint_S \operatorname{curl} \vec{F} \cdot \vec{n}\, dS = \int_C \vec{F} \cdot d\vec{r}$

Regions:



Greens Thm: $\iint_R \operatorname{div} \vec{F}\, dA = \int_C \vec{F} \cdot \vec{n}\, dr$

Solids:



Divergence Thm: $\iiint_E \operatorname{div} F\, dV = \iint_S \vec{F} \cdot \vec{n}\, dS$

In general, if Ω is a "good shape" with boundary $\partial\Omega$, then

$$\int_\Omega d\varphi = \int_{\partial\Omega} \varphi$$

Stokes Thm

not on exam.