13.6 Quadric Surfaces pt II

Recall: A quadric surface has general form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

nonexamples:

- $xyz = 0$
- $x^3 = 5$
- $\sin(y) = z$

eg. $\dfrac{x^2}{49} + \dfrac{y^2}{49} - \dfrac{z^2}{49} = 1$

Find x, y, z intercepts and xy, xz, yz traces.

x intercept: Set $y = 0$ and $z = 0$

$$\frac{x^2}{49} = 1$$

$$x^2 = 49$$

$$x = \pm 7$$

y intercept: Set $x = 0$ and $z = 0$

$$\frac{y^2}{49} = 1 \qquad y = \pm 7$$

z intercept: Set $x = 0$ $y = 0$

$$-\frac{z^2}{49} = 1$$

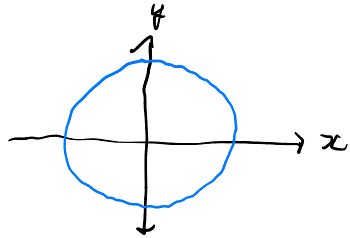
$z^2 = -49$ no solutions

$$\frac{x^2}{49} + \frac{y^2}{49} - \frac{z^2}{49} = 1$$

xy-trace: Set $z = 0$

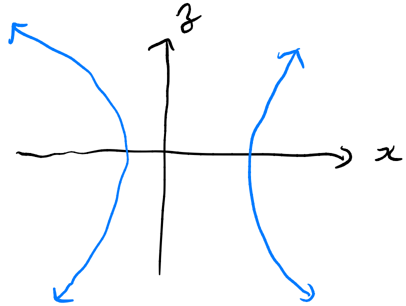
$$\frac{x^2}{49} + \frac{y^2}{49} = 1$$

$$x^2 + y^2 = 49$$



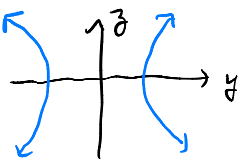
xz-trace: Set $y = 0$

$$\frac{x^2}{49} - \frac{z^2}{49} = 1$$



yz-trace: Set $x = 0$

$$\frac{y^2}{49} - \frac{z^2}{49} = 1$$

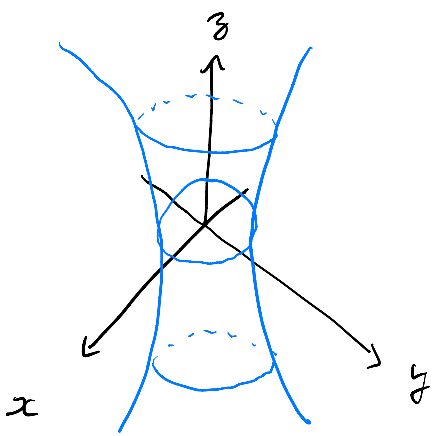


$z = k$ trace

$$\frac{x^2}{49} + \frac{y^2}{49} - \frac{k^2}{49} = 1$$

$$x^2 + y^2 = 49 + k^2$$

circle!



hyperboloid of one sheet.

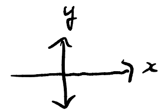
e.g. • $\frac{-x^2}{36} - \frac{y^2}{36} + \frac{z^2}{36} = 1$

x inter : $\frac{-x^2}{36} = 1$ no soln

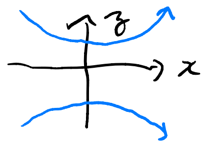
y inter : $\frac{-y^2}{36} = 1$ no soln

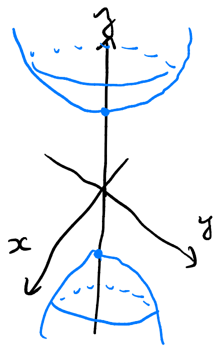
z int : $\frac{z^2}{36} = 1$ $z = \pm 6$

xy trace : $x^2 + y^2 = -36$

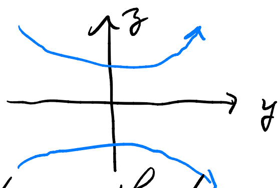


xz-trace : $-x^2 + z^2 = 36$





yz-trace : $-y^2 + z^2 = 36$



hyperboloid of two sheets.

- $$\frac{x^2}{25} + \frac{y^2}{25} - \frac{z^2}{25} = 0$$

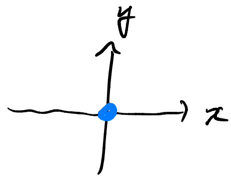
Find x, y, z inter. and xy, xz, yz traces.

x inter $\quad \frac{x^2}{25} = 0 \quad x = 0$

y inter $\quad \frac{y^2}{25} = 0 \quad y = 0$

z inter $\quad -\frac{z^2}{25} = 0 \quad z = 0$

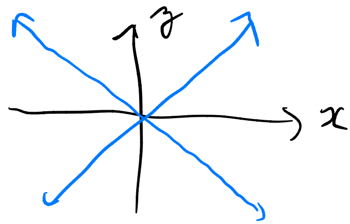
xy trace: $\quad \frac{x^2}{25} + \frac{y^2}{25} = 0$



xz trace: $\quad \frac{x^2}{25} - \frac{z^2}{25} = 0$

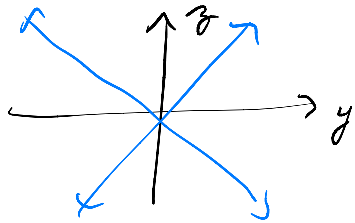
$x^2 = z^2$

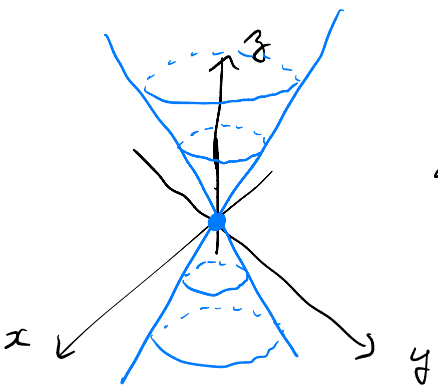
$x = \pm z$



yz trace: $\quad \frac{y^2}{25} - \frac{z^2}{25} = 0$

$y = \pm z$

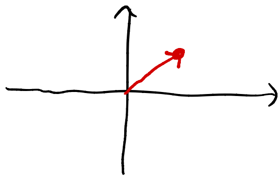




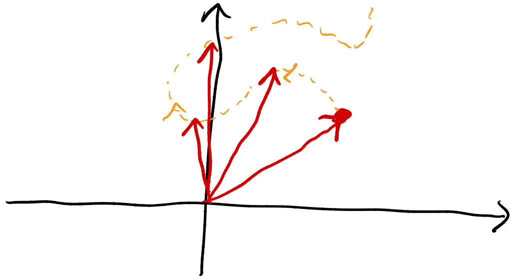
double cone

14.1 Vector valued functions

Vector



Vector functions



eg. • $r(t) = \langle 3t+1,\ 2t,\ 5 \rangle$

• $s(t) = 6t\,\hat{i} + (t^2+\cos t)\,\hat{k}$
$= \langle 6t,\ 0,\ t^2+\cos t \rangle$

• $\begin{cases} x(t) = \cos t \\ y(t) = \sin t \\ z(t) = 0 \end{cases} = \langle \cos t,\ \sin t,\ 0 \rangle$

• $f(x) = 3x^2+5$ scalar functions

e.g. Find a function

$$r(t) = \langle x(t), y(t), z(t) \rangle$$

that describes the line passing through $P(8,2,8)$ and $Q(2,9,6)$ given that $x(t) = -6t+8$.

$$r'(t) = \langle -6t'+8, \underline{7}t'+\underline{2}, \underline{-2}t'+\underline{8} \rangle = \langle 2,9,6 \rangle$$

When does $r(t) = \langle 8,2,8 \rangle$

$$-6t+8 = 8$$
$$-6t = 0$$
$$t = 0$$

$?\times(1)+8 = 6$

$? = -2$

When does $r(t) = \langle 2,9,6 \rangle$

$$-6t+8 = 2$$
$$t = 1$$

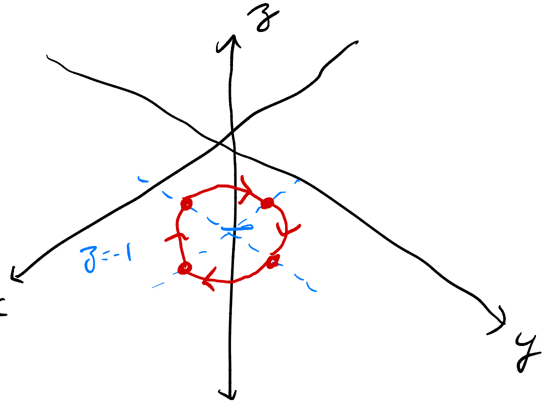
$?(1)+2 = 9$

$?(1) = 7$

$? = 7$

eg. Graph curve $r(t) = \langle -\cos t, \sin t, -1 \rangle$ from $0 \le t \le 2\pi$.

t	x	y	z
0	-1	0	-1
$\pi/2$	0	1	-1
π	1	0	-1
$3\pi/2$	0	-1	-1
2π	-1	0	-1

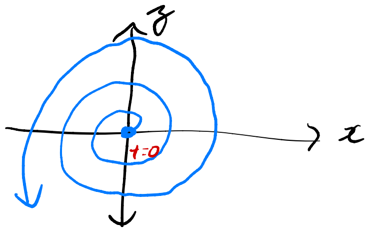


circle on the $z=-1$ plane
radius and the orientation is clockwise.

eg Graph $r(t) = \langle 3t\cos t, 3t, 3t\sin t \rangle$
$0 \le t \le 6\pi$.

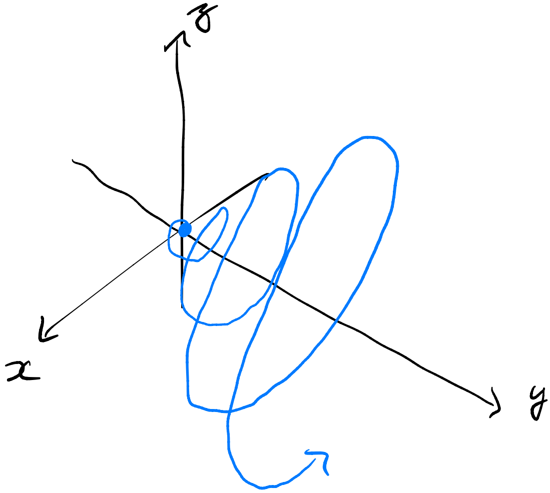
Consider only x, z coordinates

$x = 3t\cos t$
$z = 3t\sin t$



t	x	z
0		
$\pi/2$		
$\vdots$		

As t increases what happens to $y = 3t$?



eg. Find the domain of

$$r(t) = \left\langle \sqrt{16-t^2},\ \sqrt{t},\ \frac{7}{\sqrt{3+t}} \right\rangle$$

$\sqrt{16-t^2}$:

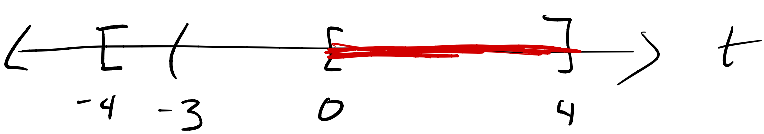
$$\begin{aligned} 16 - t^2 &\geq 0 \\ -t^2 &\geq -16 \\ t^2 &\leq 16 \\ -4 \leq t &\leq 4 \end{aligned}$$

$\sqrt{t}$: $\quad t \geq 0$

$\dfrac{7}{\sqrt{3+t}}$: $\quad 3 + t \geq 0 \qquad 3 + t \neq 0$

$\qquad t \geq -3 \qquad t \neq -3$

$\qquad t > -3$



$$0 \leq t \leq 4$$