

Lesson 2: One Sided Limits and Computing Limits Numerically

January 15, 2020

Example 1

Compute

$$\lim_{x \rightarrow 0^-} 2x + 7$$

and

$$\lim_{x \rightarrow 0^+} 2x + 7.$$

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x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$	5	6.8	6.98	6.998	—	7.002	7.02	7.2	9

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Example 2

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x	2.9	2.99	2.999	2.9999	3
$f(x)$.01616	.00166	.000167	.0000167	—

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x	2.9	2.99	2.999	2.9999	3
$f(x)$.01616	.00166	.000167	.0000167	—

Conclusion:

$$\lim_{x \rightarrow 3^-} \frac{x - 3}{x^2 - 4x - 3} = 0.$$

Example 3

Compute

$$\lim_{x \rightarrow 4^-} \frac{1}{x - 4}$$

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x	3.9	3.99	3.999	3.9999	4	4.0001	4.001	4.01	4.1
$f(x)$	-10	-100	-1000	-10000	-	10000	1000	100	10

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$$\lim_{x \rightarrow 4^+} \frac{1}{x - 4} = \infty.$$

$$\lim_{x \rightarrow 4} \frac{1}{x - 4} \text{ DNE.}$$

Example 4

For

$$f(x) = \begin{cases} x^2 + 1 & \text{for } x \leq -2 \\ 2x + 1 & \text{for } x > -2 \end{cases},$$

compute

$$\lim_{x \rightarrow -2^-} f(x) \text{ and } \lim_{x \rightarrow -2^+} f(x).$$

Example 4

For

$$f(x) = \begin{cases} x^2 + 1 & \text{for } x \leq -2 \\ 2x + 1 & \text{for } x > -2 \end{cases},$$

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x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
$f(x)$	5.41	5.04	5.00	-	-3.00	-2.98	-2.8

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Conclusion:

$$\lim_{x \rightarrow -2^-} f(x) = 5.$$

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Conclusion:

$$\lim_{x \rightarrow -2^-} f(x) = 5.$$

$$\lim_{x \rightarrow -2^+} f(x) = -3.$$

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Conclusion:

$$\lim_{x \rightarrow -2^-} f(x) = 5.$$

$$\lim_{x \rightarrow -2^+} f(x) = -3.$$

$$\lim_{x \rightarrow -2} f(x) \text{ DNE.}$$

Example 5

Compute

$$\lim_{x \rightarrow 0} 2x + 7.$$

x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$									

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Conclusion:

$$\lim_{x \rightarrow 0} 2x + 7 = 7.$$

Example 6

Compute

$$\lim_{x \rightarrow 1} 2x^2 - 3x + 1.$$

x	0	.9	.99	.999	1	1.001	1.01	1.1	2
$f(x)$									

Example 6

Compute

$$\lim_{x \rightarrow 1} 2x^2 - 3x + 1.$$

x	0	.9	.99	.999	1	1.001	1.01	1.1	2
$f(x)$	1	-.08	-.0098	-.0010	-	.0010	.0102	.12	3

Example 6

Compute

$$\lim_{x \rightarrow 1} 2x^2 - 3x + 1.$$

x	0	.9	.99	.999	1	1.001	1.01	1.1	2
$f(x)$	1	-.08	-.0098	-.0010	-	.0010	.0102	.12	3

Conclusion:

$$\lim_{x \rightarrow 1} 2x^2 - 3x + 1 = 0.$$

Example 7

Compute

$$\lim_{x \rightarrow -1} \frac{1}{(x+1)^2}.$$

x	-2	-1.1	-1.01	-1.001	-1	-.999	-.99	-.9	0
$f(x)$									

Example 7

Compute

$$\lim_{x \rightarrow -1} \frac{1}{(x+1)^2}.$$

x	-2	-1.1	-1.01	-1.001	-1	-.999	-.99	-.9	0
$f(x)$	1	100	10000	1000000	-	1000000	10000	100	1

Example 7

Compute

$$\lim_{x \rightarrow -1} \frac{1}{(x+1)^2}.$$

x	-2	-1.1	-1.01	-1.001	-1	-.999	-.99	-.9	0
$f(x)$	1	100	10000	1000000	-	1000000	10000	100	1

Conclusion:

$$\lim_{x \rightarrow -1} \frac{1}{(x+1)^2} = \infty.$$

Example 8

Compute

$$\lim_{x \rightarrow 0} \frac{1}{x}.$$

x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$									

Example 8

Compute

$$\lim_{x \rightarrow 0} \frac{1}{x}.$$

x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$	-1	-10	-100	-1000	-	1000	100	10	1

Example 8

Compute

$$\lim_{x \rightarrow 0} \frac{1}{x}.$$

x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$	-1	-10	-100	-1000	-	1000	100	10	1

Conclusion:

$$\lim_{x \rightarrow 0} \frac{1}{x} \text{ DNE.}$$

Example 9

Compute

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x}.$$

x	-1	-0.1	-0.01	-0.001	0	.001	.01	.1	1
$f(x)$									

Example 9

Compute

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x}.$$

x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$.842	.998	1.000	1.000	—	1.000	1.000	.998	.842

Example 9

Compute

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x}.$$

x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$.842	.998	1.000	1.000	—	1.000	1.000	.998	.842

Conclusion:

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1.$$

Example 10

Compute

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x}.$$

x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$									

Example 10

Compute

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x}.$$

x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$.46	.05	.005	.001	-	-.001	-.005	-.05	-.46

Example 10

Compute

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x}.$$

x	-1	-.1	-.01	-.001	0	.001	.01	.1	1
$f(x)$.46	.05	.005	.001	-	-.001	-.005	-.05	-.46

Conclusion:

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x} = 0.$$