Problem 1. Use the Second Derivative Test to find the local extrema of

$$f(x) = xe^{-x}.$$

Solution: We first need to find the critical numbers, places where $f'(x)$ is zero or does not exist. By the product rule and chain rule, we have that

$$f'(x) = e^{-x} - xe^{-x} = e^{-x}(1 - x).$$

Hence the only critical number is $x = 1$.

Next we need to classify the critical number $x = 1$ by the Second Derivative Test. The product and chain rules again give:

$$f''(x) = -e^{-x} - e^{-x} + xe^{-x} = e^{-x}(x - 2).$$

We therefore have that $f''(1) < 0$, hence f has a local maximum at $(1, f(1)) = (1, e^{-1})$.

Problem 2. Find the absolute maximum AND absolute minimum of

$$f(x) = \frac{\ln(2x)}{x}$$

on $[1, e]$.

Solution: We first find the critical numbers. By the quotient and chain rules, we have that

$$f'(x) = \frac{\frac{2x}{2x} - \ln(2x)1}{x^2} = \frac{1 - \ln(2x)}{x^2}.$$

Now $f'(x)$ does not exist when $x = 0$, and $f'(x)$ is zero when $x = e/2$. Since 0 is not in the domain of $f(x)$, the only critical number is $e/2$. Since

$$\begin{aligned} f(1) &= \ln(2) \approx 0.693 \\ f(e/2) &= 2/e \approx 0.736 \\ f(e) &= \ln(2e)/e \approx 0.623, \end{aligned}$$

we conclude that f has an absolute maximum at $(e/2, 2/e)$ and an absolute minimum at $(e, \ln(2e)/e)$.