Problem 1. Determine the location and type (hole, vertical asymptote, or jump) of any discontinuities:

$$f(x) = \frac{(x+2)^2}{x^2 - x - 6}$$

<u>**Solution**</u>: Discontinuities of rational functions occur at x-values which make the denominator zero. Since

$$x^2 - x - 6 = (x+2)(x-3),$$

the only discontinuities are at $x = -2$ and $x = 3$. Since

$$\frac{(x+2)^2}{x^2 - x - 6} = \frac{x+2}{x-3},$$

the "bad spot" in the denominator cancels for $x = -2$ but not for $x = 3$. Hence there is a hole at $x = -2$ and a vertical asymptote at $x = 3$.

Problem 2. Use the limit definition of the derivative to compute the derivative of

$$f(x) = x^2 + 1.$$

Solution:

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{((x+h)^2 + 1) - (x^2 + 1)}{h} \\
&= \lim_{h\to 0} \frac{(x^2 + 2hx + h^2 + 1) - (x^2 + 1)}{h} \\
&= \lim_{h\to 0} \frac{2hx + h^2}{h} \\
&= \lim_{h\to 0} 2x + h \\
&= 2x.
\end{aligned}$$