

Problem 1. Use the basic rules of differentiation to compute the derivative of

$$f(x) = \sqrt{x} \left(3x^2 - \frac{1}{x} \right).$$

Solution:

$$\begin{aligned} f'(x) &= \frac{d}{dx} \left[\sqrt{x} \left(3x^2 - \frac{1}{x} \right) \right] \\ &= \frac{d}{dx} \left[x^{1/2} (3x^2 - x^{-1}) \right] \\ &= \frac{d}{dx} \left[3x^{5/2} - x^{-1/2} \right] \\ &= 3 \frac{d}{dx} \left[x^{5/2} \right] - \frac{d}{dx} \left[x^{-1/2} \right] \\ &= \frac{15}{2} x^{3/2} + \frac{1}{2} x^{-3/2}. \end{aligned}$$

Problem 2. The position of a particle on a straight line is given by

$$s(t) = \frac{t^3}{3} - t^2 + t.$$

What is the particle's position when its velocity is zero?

Solution: The velocity of the particle is

$$v(t) = t^2 - 2t + 1.$$

Hence the velocity is zero when

$$\begin{aligned} t^2 - 2t + 1 &= 0 \\ (t - 1)^2 &= 0 \\ t &= 1. \end{aligned}$$

The position of the particle when the velocity is zero is

$$s(1) = \frac{1}{3}.$$