Problem 1. Find the critical numbers of

$$f(x) = \frac{x}{x^2 + 2}.$$

Solution: Using the quotient rule, we compute that

$$f'(x) = \frac{(x^2+2)1 - x(2x)}{(x^2+2)^2} = \frac{-x^2+2}{(x^2+2)^2}.$$

The critical numbers are where the derivative does not exist or is zero. Note that $f'(x)$ exists everywhere because $(x^2+2)^2$ can never be zero. We have that $f'(x) = 0$ when the numerator is equal to zero, that is, when $-x^2+2 = 0$. Hence the critical numbers are

$$x = -\sqrt{2} \quad \text{and} \quad x = \sqrt{2}.$$

Problem 2. Find the local maximums and local minimums of

$$f(x) = x^2 e^x.$$

Solution: By the product rule,

$$f'(x) = 2xe^x + x^2e^x = xe^x(2+x).$$

Since $f'(x)$ exists everywhere, the critical numbers are where this derivative is zero: $x = 0$ and $x = -2$. Applying a number line test to $f'(x)$, we find that $f'(x) > 0$ on $(-\infty, -2) \cup (0, \infty)$ and $f'(x) < 0$ on $(-2, 0)$. It follows from the First Derivative Test that f has

$$\text{a local maximum at } (-2, f(-2)) = (-2, 4e^{-2}),$$

and

$$\text{a local minimum at } (0, f(0)) = (0, 0).$$