Lesson 11: Sections 3.6 and 3.7 (Part I)

Derivatives as Rates of Change

- average rate of change for $f(x)$ on interval $[x, x+h]$ is

$$\frac{f(x+h)-f(x)}{h},$$

the slope of the secant line

- instantaneous rate of change for $f(x)$ at x is

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x),$$

the slope of the tangent line at x

- If $p(t)$ gives the position of an object at time t, then the rate of change of position is velocity, and the rate of change of velocity is acceleration:

$p(t)$ = position

$v(t) = p'(t)$ = velocity

$a(t) = v'(t) = p''(t)$ = acceleration

- The sign of $v(t)$ tells us the direction of movement:

$v(t) > 0$: moving in positive direction

$v(t) = 0$: stationary

$v(t) < 0$: moving in negative direction

- Speed $= |v(t)|$

The Chain Rule

- Use the chain rule to take the derivative of a composition of functions:

If $y = f(g(x))$,

$$\frac{dy}{dx} = \frac{dy}{dg} \cdot \frac{dg}{dx}$$

$$y' = f'(g(x)) \cdot g'(x)$$

- "The derivative of the outside with the inside left alone
times
the derivative of the inside"

Practice Problems

1. An object thrown vertically upward from the ground at a velocity of $20\frac{m}{s}$ reaches a height of $s(t) = -2t^2 + 20t$ meters in t seconds.

(a) Find $v(t)$ and graph it.

(b) When does the object reach its highest point?

(c) What is the maximum height of the object?

(d) When does the object strike the ground?

(e) With what velocity does the object strike the ground?

(f) On what intervals is the speed increasing?

2. Find $\frac{d}{dx}\left(\frac{2}{(x^3+9)^7}\right)$