Lesson 13: Section 3.7 (Part 2)

Recall:

$$\frac{d}{dx}\left(f(g(x)\right) = f'(g(x)) \cdot g'(x)$$

- If more than one function is composed, multiple applications of the chain rule give a result following the same pattern: at each step take the derivative of the outer function leaving the inside alone:

$$\frac{d}{dx}\left(f(g(h(x)))\right)$$

$$= f'(g(h(x)) \cdot g'(h(x)) \cdot h'(x)$$

Example:

$$\frac{d}{dx}\left(\sin(\tan^2(e^x))\right)$$

$$= \cos(\tan^2(e^x)) \cdot 2\tan(e^x) \cdot \sec^2(e^x) \cdot e^x$$

- Sometimes you'll need to use the chain rule with the product / quotient rule.

Example:

$$\frac{d}{dx}\left(\sin(2x)\cdot e^{3x}\right)$$

$$= \frac{d}{dx}\left(\sin(2x)\right)\cdot e^{3x} + \sin(2x)\cdot \frac{d}{dx}\left(e^{3x}\right)$$

$$= \cos(2x)\cdot 2 \cdot e^{3x} + \sin(2x)\cdot e^{3x}\cdot 3$$

Practice Problems

1. If $f(x)$ and $g(x)$ are such that

$$f(0) = 2 \qquad g(0) = 4$$
$$f'(0) = 3 \qquad g'(0) = 5$$

find

(a) $h'(0)$, where $h(x) = f(2x)g(3x)$

(b) $j'(0)$, where $j(x) = \dfrac{f(2x)}{g(3x)}$.

(c) $k'(0)$, where $k(x) = (f(2x))^2$

2. Use the chain rule and the fact that $x = e^{\ln(x)}$ to find

$$\frac{d}{dx}(\ln(x)).$$

3. A formula for $\frac{d}{dx}(a^x)$ where $a>0$ is a constant.

(a) Find a function $f(x)$ satisfying

(i) $a^x = e^{f(x)}$

(ii) you know how to compute $f'(x)$

(b) Use the chain rule to produce the formula:

$$\frac{d}{dx}(a^x) = \frac{d}{dx}\left(e^{f(x)}\right) =$$

(c) Since we know $\frac{d}{dx}(e^x) = e^x$, check your formula by plugging in $a = e$.

4. (Challenge)
Use the idea from 2 and your result from 3 to find

$$\frac{d}{dx}(\log_a(x)) \text{ for } a>0.$$