Lesson 14: Section 3.8

Implicit Differentiation

- If y is a function of x, we can compute derivatives of the form

$$\frac{d}{dx}\left(\begin{array}{c}\text{expression}\\\text{involving}\\x \text{ and } y\end{array}\right)$$

Examples:

- $\frac{d}{dx}\left(\sec(y)\right) = \sec(y)\cdot\tan(y)\cdot\frac{dy}{dx}$
- $\frac{d}{dx}\left(x^2+y^3\right) = 2x + 3y^2\frac{dy}{dx}$
- $\frac{d}{dx}\left(x e^y\right) = 1\cdot e^y + x\cdot e^y\cdot\frac{dy}{dx}$

From the Chain Rule!

- Goal: Given an equation like $xy = \sin(xy)$ where y is a function of x, find either
  (1) an explicit form of $\frac{dy}{dx}$, this is an equation with $\frac{dy}{dx}$ isolated on one side of the equation
  (2) an implicit form of $\frac{dy}{dx}$, this is an equation involving $\frac{dy}{dx}$ where $\frac{dy}{dx}$ is not isolated

- When studying related rates next week, we will see situations where

$$t = \text{independent variable}$$
$$x = \text{function of } t$$
$$y = \text{function of } t$$

In a setting like this we can compute derivatives of the form

$$\frac{d}{dt}\left(\begin{matrix}\text{expression involving} \\ t,\ x, \text{ and } y\end{matrix}\right)$$

<u>Example</u>: $\frac{d}{dt}\left(x^2+y^3\right) = 2x\frac{dx}{dt} + 3y^2\frac{dy}{dt}$

(Chain Rule!

<u>Lesson</u>: Keep track of which variable you are differentiating with respect to!

Practice Problems

1. Find $\frac{dy}{dx}$ by implicit differentiation:

$$e^y \sin(x) = x + xy$$

2. Find the equation of the tangent to the curve defined by

$$e^{x/y} = 3x - y + 2$$

at the point $(0, 1)$.