Lesson 15: Section 3.9

Derivatives of Logarithmic Functions

- Knowing $\frac{d}{dx}(e^x) = e^x$, we can use implicit differentiation to get:
  - $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$
  - $\frac{d}{dx}(a^x) = a^x \cdot \ln(a)$
  - $\frac{d}{dx}(\log_a(x)) = \frac{1}{x \cdot \ln(a)}$
- Before taking the derivative of a logarithmic function, try simplifying using log rules (See Lesson 2 Section 1.3 to review)

Logarithmic Differentiation

- To take the derivative of $y = f(x)^{g(x)}$ use logarithmic differentiation:

  Both base and exponent change with x

1. Take ln of both sides and simplify

$$\ln(y) = \ln\left(f(x)^{g(x)}\right)$$

$$\ln(y) = g(x) \cdot \ln(f(x))$$

2. Find $\frac{dy}{dx}$ using implicit differentiation:

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}\left(g(x) \cdot \ln(f(x))\right)$$

$$\frac{dy}{dx} = y \cdot \frac{d}{dx}\left(g(x) \cdot \ln(f(x))\right)$$

- You can also use logarithmic differentiation to compute the derivative of complicated functions like

$$y = (x^7 - 83)^{79} \cdot \sqrt{x^8 - \sin(x)}.$$

Practice Problems

1. Use the properties of logarithms to simplify the following function first, then differentiate.

$$y = 2\ln\left(\frac{x\sqrt[3]{x^2+9}}{\sin(x)}\right)$$

2. $f(x)$ is a function whose tangent line at $x = \frac{\pi}{4}$ is $y = 3x - \frac{3\pi}{4} + e$. Find

$$\frac{d}{dx}\left[f(x)^{\tan(x)}\right]_{x=\frac{\pi}{4}}.$$

(Hint: Use logarithmic differentiation.)