

Lesson 16: Section 3.10

Derivatives of Inverse Functions

• You can use implicit differentiation to find the derivative of an inverse function:

$$y = f(x)$$

$$f^{-1}(y) = x$$

$$(f^{-1})'(y) \cdot \frac{dy}{dx} = 1$$

$$(f^{-1})'(y) = \frac{1}{f'(x)}$$

Example: If $f'(0) = 2$ and $(0,1)$ is on the graph of $f(x)$, then

$$(f^{-1})'(1) = \frac{1}{f'(0)} = \frac{1}{2}.$$

• This can be used to find the derivatives of inverse trig functions:

$$- \frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$- \frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$- \frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

Practice Problems

1. Find $(f^{-1})'(2)$ if $f(x) = x^3 + x^2$.
2. Find a function whose derivative is $\frac{3}{\sqrt{1-9x^2}}$.