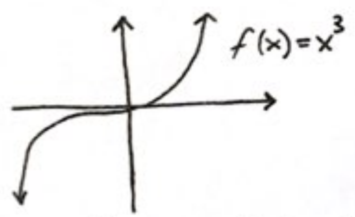
Lesson 2: Section 1.3

Inverse Functions

- $y = f(x) \iff f^{-1}(y) = x$
- f has an inverse only if it is one-to-one or equivalently, passes the horizontal line test

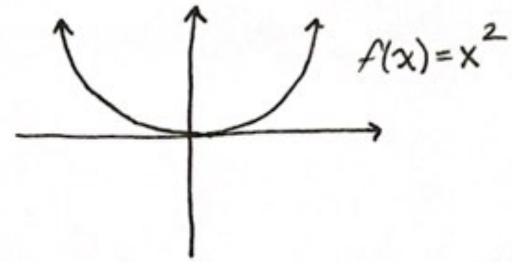
  every horizontal line crosses the graph at most once

Example:



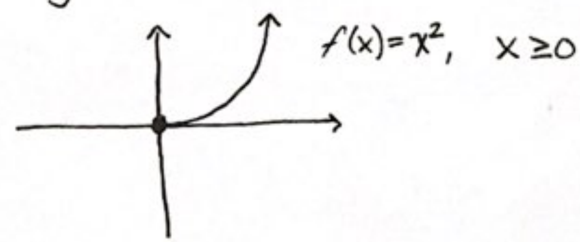
Passes horizontal line test so $f(x)=x^3$ has an inverse

$f^{-1}(x) = x^{1/3} = \sqrt[3]{x}$



Fails horizontal line test

Restricting the domain fixes this:



Passes horizontal line test so has an inverse, $f^{-1}(x) = x^{1/2} = \sqrt{x}$.

- graphs of $f(x)$ and $f^{-1}(x)$ are reflections of one another over the line $y=x$
- Domain of $f(x)$ = Range of $f^{-1}(x)$

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Exponential and Logarithmic Functions

- These functions are inverses of one another:

$$y = b^x \iff x = \log_b(y)$$

if $f(x) = b^x$, then $f^{-1}(x) = \log_b(x)$

Domain $f(x) = (-\infty, \infty) =$ Range $f^{-1}(x)$

Range $f(x) = (0, \infty) =$ Domain $f^{-1}(x)$

Useful Properties of the Logarithm

- $\log_b(xy) = \log_b(x) + \log_b(y)$ (if $x, y > 0$)
- $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$ (if $x, y > 0$)
- $\log_b(x^c) = c \log_b(x)$ (if $x > 0$)
- $\log_b(1) = 0$
- $\log_b(b) = 1$
- $b^{\log_b(x)} = x$
- $\log_b(b^x) = x$

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Practice Problems

1. If $f(x) = \sqrt{x-5}$, $x \geq 5$ what is $f^{-1}(x)$? What is the domain of $f^{-1}(x)$? Sketch a graph of f and f^{-1}.

2. If $\log_b(x) = 5$ and $\log_b(y) = 0.52$, find $\log_b\left(\frac{\sqrt[4]{x}}{y^3}\right)$.