Lesson 20: Sections 4.2, 4.3 (Part 1)

- **Rolle's Theorem:** If $f(x)$ is continuous on $[a,b]$, differentiable on (a,b), and $f(a)=f(b)$, then there is at least one value of x, call it c, $a<c<b$, where

  $f'(c)=0$.

- **Mean Value Theorem:** If $f(x)$ is continuous on $[a,b]$, differentiable on (a,b), then there is at least one value c, $a<c<b$, such that

$$f'(c)=\frac{f(b)-f(a)}{b-a}.$$

- MVT says: somewhere between a and b the tangent line is parallel to the secant line through $(a,f(a))$, $(b,f(b))$.

- If $f(a)=f(b)$, then the MVT becomes Rolle's Theorem.

Test for Intervals of Increase/Decrease

If f is continuous and differentiable on an interval I:

- $f'(x)>0$ on $I \Longrightarrow f$ is increasing on I
- $f'(x)<0$ on $I \Longrightarrow f$ is decreasing on I

First Derivative Test

If c is a critical point of f:

- f has a local max at c if f' changes from $+$ to $-$ at c
- f has a local min at c if f' changes from $-$ to $+$ at c
- f has no extreme value at c if f' doesn't change signs at c

Fact:

- Suppose f is continuous on an interval I that contains <u>exactly one</u> local extreme at c.
  - If a local max occurs at c, then $f(c)$ is the absolute max on I.
  - If a local min occurs at c, then $f(c)$ is the absolute min on I.

Practice Problems

1. Determine whether the Mean Value Theorem applies to $f(x) = \ln(2x)$ on $[1, e]$. If so, find all points c guaranteed by the MVT.

2. Find the intervals on which $f(x) = \tan^{-1}\left(\frac{x}{x^2+3}\right)$ is increasing and decreasing. Classify the critical points of f.