1. $f(x) = \ln(2x)$ is continuous on $[1, e]$ and differentiable on $(1, e)$, so MVT applies.

Now

$$\frac{f(e)-f(1)}{e-1} = \frac{\ln(2e)-\ln(2)}{e-1} = \frac{\ln(e)}{e-1} = \frac{1}{e-1}$$

$$f'(c) = \frac{1}{2c} \cdot 2 = \frac{1}{c}$$

Hence $f'(c) = \frac{f(e)-f(1)}{e-1}$

only when $c = e-1$.

2. $f(x) = \tan^{-1}\left(\frac{x}{x^2+3}\right)$

$$f'(x) = \frac{1}{1+\left(\frac{x}{x^2+3}\right)^2} \cdot \frac{d}{dx}\left(\frac{x}{x^2+3}\right)$$

$$= \frac{(x^2+3)^2}{(x^2+3)^2+x^2} \cdot \frac{(x^2+3)\cdot 1 - x(2x)}{(x^2+3)^2}$$

$$= \frac{3-x^2}{(x^2+3)^2+x^2}$$

always >0

- Critical Points: $x = \pm\sqrt{3}$
- The sign of $f'(x)$ is the same as the sign of $3-x^2$, so:
  - $3-x^2<0$ on $(-\infty, -\sqrt{3})$, $(\sqrt{3}, \infty)$, hence f decreasing on these intervals
  - $3-x^2>0$ on $(-\sqrt{3}, \sqrt{3})$, hence f increasing on this interval
- First Derivative Test says f has:

  local min at $x=-\sqrt{3}$

  local max at $x=\sqrt{3}$