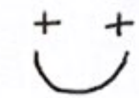
Lesson 21: Section 4.3 (Part 2)

Recall:

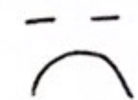
- $f'(x) > 0 \iff f$ increasing
- $f'(x) < 0 \iff f$ decreasing
- **First Derivative Test:**
  If c is a critical number of f, then f has
  - a local max at c if f' changes from $+$ to $-$
  - a local min at c if f' changes from $-$ to $+$
  - neither max nor min if f' doesn't change signs.

Test for Intervals of Concavity

- $f''(x) > 0 \iff f$ concave up



- $f''(x) < 0 \iff f$ concave down



- f has an inflection point at $x=c$ if f'' changes signs at $x=c$.

Second Derivative Test

If c is a critical point of f, then:

- f has a local min at $x=c$ if $f''(c) > 0$
- f has a local max at $x=c$ if $f''(c) < 0$
- Must use First Derivative Test if $f''(c) = 0$.

Practice Problems

1. Let $f(x) = -3x^5 + 5x^3$.

   (a) Find the critical points of f.

   (b) Determine the intervals of concavity of f.

   (c) What does the Second Derivative Test say about the critical points?

   (d) Determine the inflection points of f.

2. Find and classify the critical points of $f(x) = x^2 e^x$ and $g(x) = x^3 e^x$ using the Second Derivative Test if possible. How does this process differ for these two functions?