Lesson 22: Section 4.4 (Part 1)

Goal: Produce an accurate sketch of the graph of $f(x)$.

Information Gathered from f:

- Domain of f
- Does f have y-axis symmetry or origin symmetry? (Tomorrow's lesson will emphasize this)
- x-intercepts (Find when $f(x)=0$)
- y-intercept (The point $(0, f(0))$)
- Horizontal, vertical, and slant asymptotes.

CAUTION: The lecture video incorrectly explains how to find slant asymptotes. There is a slant asymptote when the numerator degree is one larger than the denominator degree. Use polynomial long division and throw away the remainder.

Example:

$$f(x) = \frac{x^2+15}{5x+1}$$

$$\begin{array}{r} \frac{1}{5}x - \frac{1}{25} \\ 5x+1 \,\overline{)\, x^2+15} \\ -\left(x^2+\frac{1}{5}x\right) \\ \hline -\frac{1}{5}x+15 \\ -\left(-\frac{1}{5}x-\frac{1}{25}\right) \\ \hline \frac{376}{25} \end{array}$$

← Remainder

Slant Asymptote: $y = \frac{1}{5}x - \frac{1}{25}$

Information Gathered from f'

- Intervals where f is increasing / decreasing ($f'>0$ / $f'<0$)
- Local extrema of f (Find the critical numbers and classify using the 1ST Derivative Test)

Information Gathered from f''

- Intervals where f is concave up / concave down ($f''>0$ / $f''<0$)
- Inflection Points of f (where f'' changes signs)

Putting it all together

- General Shape:
  - draw asymptotes
  - graph shapes:
    - concave up, decreasing: ∪
    - concave up, increasing: ∪
    - concave down, increasing: ∩
    - concave down, decreasing: ∩
- More Precision:
  - Plot the points you found (x-intercepts, y-intercept, extrema, inflection points)

Practice Problems

1. Sketch the graph of

$$f(x) = \frac{x}{x^2+1}$$

given that

$$f'(x) = \frac{1-x^2}{(x^2+1)^2}$$

$$f''(x) = \frac{2x^3-6x}{(x^2+1)^3}.$$

2. Find all asymptotes for

$$f(x) = \frac{4x^2-3}{x-4}.$$