Lesson 23: Section 4.4 (Part 2)

Goal: Produce an accurate sketch of the graph of f.

Symmetry:

- Even Functions: A function f is even if $f(x) = f(-x)$ for every x.
- The graph of an even function is symmetric about the y-axis (if we "fold" along the y-axis the graph lines up)

Examples: • $f(x) = x^n$, n-even
• $f(x) = \cos(x)$

- Odd Functions: A function f is odd if $f(-x) = -f(x)$ for every x.
- The graph of an odd function is symmetric about the origin (reflecting a point on the graph across the origin (equivalently, reflect across the y-axis then the x-axis) gives a point on the graph.

Examples: • $f(x) = x^n$, n-odd
• $f(x) = \sin(x)$

Note: Many functions are neither even nor odd! For example, $y = (x+1)^2$.

Practice Problems

1. Sketch a graph of $f(x)$ given that $f'(x) = x^2 e^{-2x}$. Find intervals where f is increasing/decreasing as well as intervals where f is concave up/concave down.

2. If $y = f(x)$ is an even function and $y = g(x)$ is an odd function, what can you say about the symmetry of the following functions?

(a) $F(x) = \frac{f(x)}{g(x)}$

(b) $G(x) = 2 \cdot f(x)$

(c) $H(x) = f(x) + 3$

(d) $R(x) = g(x) + 3$

(e) $S(x) = (g(x))^2$

(f) $T(x) = \sin(g(x))$