

1. Distance from $(x, 2-x^2)$ to the origin is

$$f(x) = \sqrt{x^2 + (2-x^2)^2}.$$

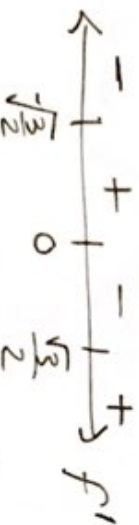
We will find the x -values that minimize this by finding the x -values that minimize

$$F(x) = (f(x))^2 = x^2 + (2-x^2)^2.$$

The interval of interest is $(-\infty, \infty)$.

$$\begin{aligned} F'(x) &= 2x + 2(2-x^2) \cdot (-2x) \\ &= 2x(-3+2x^2) \end{aligned}$$

Crit Pts: $x=0, x=\pm\sqrt{\frac{3}{2}}$



Relative minimums at $x = \pm\sqrt{\frac{3}{2}}$.

Since $f(\sqrt{\frac{3}{2}}) = f(-\sqrt{\frac{3}{2}})$, both of these give min. distance.

The closest points are $(\pm\sqrt{\frac{3}{2}}, \frac{1}{2})$.

L24 P3

2. We want to maximize

$$V = \pi r^2 h$$

Subject to the constraint

$$h + 2\pi r = 108$$

$$h = 108 - 2\pi r$$

$$\begin{aligned} V(r) &= \pi r^2 (108 - 2\pi r) \\ &= 108\pi r^2 - 2\pi^2 r^3 \end{aligned}$$

Interval of interest: $[0, \frac{54}{\pi}]$

$$V'(r) = 216\pi r - 6\pi^2 r^2$$

Crit Pts: $r=0, r=\frac{36}{\pi}$

$$V''(r) = 216\pi - 12\pi^2 r$$

$$V''(\frac{36}{\pi}) < 0 \text{ so rel. max at } \frac{36}{\pi}$$

Since this is the only critical point on the interval of interest, it is also the absolute max.

Answer: $V(\frac{36}{\pi}) \approx 14.851 \text{ in}^3$