1. Distance from $(x, 2-x^2)$ to the origin is

$$f(x) = \sqrt{x^2 + (2-x^2)^2}.$$

We will find the x-values that minimize this by finding the x-values that minimize

$$F(x) = (f(x))^2 = x^2 + (2-x^2)^2.$$

The interval of interest is $(-\infty, \infty)$.

$$F'(x) = 2x + 2(2-x^2)\cdot(-2x)$$
$$= 2x(-3+2x^2)$$

Crit Pts: $x=0$, $x = \pm\sqrt{\frac{3}{2}}$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ & -\sqrt{\frac{3}{2}} & & 0 & & \frac{\sqrt{3}}{2} & \end{array} \quad f'$$

Relative minimums at $x = \pm\sqrt{\frac{3}{2}}$.

Since $f\left(\sqrt{\frac{3}{2}}\right) = f\left(-\sqrt{\frac{3}{2}}\right)$, bothe of these give min. distance. The closest points are $\left(\pm\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$.

L24 P3

2. We want to maximize

$$V = \pi r^2 h$$

Subject to the constraint

$$h + 2\pi r = 108$$
$$h = 108 - 2\pi r$$

$$V(r) = \pi r^2 (108 - 2\pi r)$$
$$= 108\pi r^2 - 2\pi^2 r^3$$

Interval of interest: $\left[0, \frac{54}{\pi}\right]$

$$V'(r) = 216\pi r - 6\pi^2 r^2$$

Crit Pts: $r=0$, $r = \frac{36}{\pi}$

$$V''(r) = 216\pi - 12\pi^2 r$$

$V''\left(\frac{36}{\pi}\right) < 0$ so rel. max at $\frac{36}{\pi}$

Since this is the only critical point on the interval of interest, it is also the absolute max.

Answer: $V\left(\frac{36}{\pi}\right) \approx 14.851 \text{ in}^3$