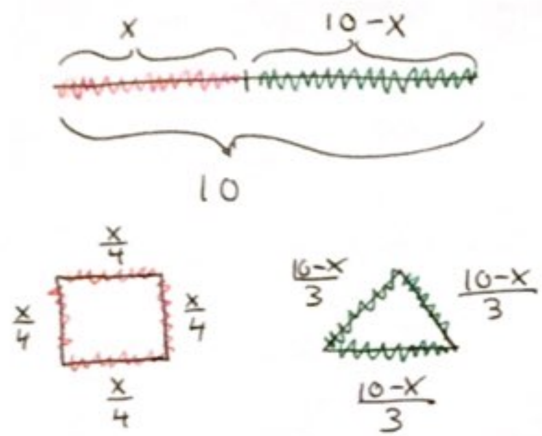
1.



<u>Objective Function:</u> $A(x) = \underbrace{\frac{x^2}{16}}_{\text{Area of Square}} + \underbrace{\frac{\sqrt{3}}{4} \cdot \frac{(10-x)^2}{9}}_{\text{Area of Triangle}}$

<u>Interval of Interest:</u> $[0,10]$

$$A'(x) = \frac{x}{8} - \frac{\sqrt{3}(10-x)}{18}$$
$$= \left(\frac{1}{8} + \frac{\sqrt{3}}{18}\right)x - \frac{5\sqrt{3}}{9}$$

<u>Critical Point:</u> $x = \dfrac{\frac{5\sqrt{3}}{9}}{\left(\frac{1}{8}+\frac{\sqrt{3}}{18}\right)} \approx 4.3496$

Since $A''(x) > 0$, this is a minimum.
This is the only critical point in the interval, hence it is an absolute minimum.
Since $A(0) = \frac{25}{3\sqrt{3}} \approx 4.81125$ and $A(10) = \frac{25}{4} = 6.25$, the max occurs if all wire is used for the square.

L25 P2

2.

<u>Constraint:</u>

$$\underbrace{2r + 2l}_{\substack{\text{rectangulor perimeter} \\ \text{(excluding top)}}} + \underbrace{\pi r}_{\substack{\text{half} \\ \text{circumference}}} = 30$$

or equivalently, $l = \dfrac{30 - (2+\pi)r}{2}$

<u>Objective Function:</u>

$$A = \underbrace{2rl}_{\text{Area of Rectangle}} + \underbrace{\tfrac{1}{2}\pi r^2}_{\text{Half Area of Circle}}$$
$$= 2r\left(\frac{30-(2+\pi)r}{2}\right) + \frac{1}{2}\pi r^2$$
$$= 30r - (2+\pi)r^2 + \frac{1}{2}\pi r^2$$

<u>Interval of Interest:</u> $\left[0, \frac{30}{2+\pi}\right]$

$$A'(r) = 30 - (4+2\pi)r + \pi r$$
$$= 30 - (4+\pi)r$$

<u>Critical Point:</u> $r = \frac{30}{4+\pi}$

Since this is the only critical point in the interval and $A'' < 0$, this gives an absolute max.

<u>Best Dimensions:</u> $\frac{60}{4+\pi} \times \dfrac{30 - (2+\pi)\left(\frac{30}{4+\pi}\right)}{2}$

or about 8.401 ft $\times$ 4.201 ft.