Lesson 27: Section 4.6

Linear Approximation and Differential

- The tangent line to $f(x)$ at $x=a$ is close to the graph of $f(x)$ near a:

$$\underbrace{y = f(a) + f'(a)(x-a)}_{\text{Equation of tangent line}} \approx f(x)$$

if x is close to a

- $L(x) = f(a) + f'(a)(x-a)$ is the linear approximation of $f(x)$ at $x=a$

- $f'(a)(x-a)$ is the differential. It estimates the change in $f(x)$ as x changes from a.

Using Linear Approximations to Estimate Values

1. Choose a function $f(x)$ so that the value you want to estimate is $f(c)$ for some c
2. Choose a number a near c such that $f(a)$ is easy to compute.
3. $f(c) \approx \underbrace{f(a) + f'(a)(c-a)}_{L(c)}$

- The error in the approximation is

$$|\text{approx} - \text{actual}| = |L(c) - f(c)|$$

- The percentage error is

$$100 \cdot \frac{|\text{approx} - \text{actual}|}{\text{actual}} = 100 \cdot \frac{|L(c) - f(c)|}{f(c)}$$

Practice Problems

1 (a) Use linear approximations to justify the estimate

$$(1+x)^n \approx 1 + nx$$

for any number n.

(b) What number a must x be close to in order for the estimate to be good?

(c) Find all x-values so that the percentage error is $\leq 5\%$. in the estimate

$$(1+x)^2 \approx 1+2x$$

2. Use linear approximations to estimate

(a) $\tan(0.2)$

(b) $\sqrt{65}$

(c) $\ln(1.1)$