

1. (a) and (b)

For  $a=0$ , the linear approx of  $f(x) = (1+x)^n$  at  $a=0$  is

$$\begin{aligned} L(x) &= f(a) + f'(a)(x-a) \\ &= 1 + n \cdot (x-0) \\ &= 1 + nx \end{aligned}$$

$$\begin{aligned} \text{(c) Percentage Error} &= 100 \cdot \frac{|x^2+2x+1-2x+1|}{x^2+2x+1} \\ &= \frac{100x^2}{x^2+2x+1} \end{aligned}$$

$$\begin{aligned} \frac{100x^2}{x^2+2x+1} &= 5 \iff 19x^2 - 2x - 1 = 0 \\ &\iff x = \frac{1 \pm 2\sqrt{5}}{19} \end{aligned}$$

The percentage error is  $\leq 5\%$  if  $x$  in  $\left[\frac{1-2\sqrt{5}}{19}, \frac{1+2\sqrt{5}}{19}\right]$ .

L27 P3

2. (a)

$\tan(0.2) = f(0.2)$   
where  $f(x) = \tan(x)$ .

$$\begin{aligned} \tan(0.2) &\approx f(0) + f'(0)(0.2-0) \\ &= 0 + 1(0.2) \\ &= 0.2 \end{aligned}$$

(b)  $\sqrt{65} = f(65)$   
where  $f(x) = \sqrt{x}$ .

$$\begin{aligned} \sqrt{65} &\approx f(64) + f'(64)(65-64) \\ &= 8 + \frac{1}{16}(1) \\ &= \frac{129}{16} \end{aligned}$$

(c)  $\ln(1.1) = f(1.1)$   
where  $f(x) = \ln(x)$   
 $\ln(1.1) \approx f(1) + f'(1)(1.1-1)$   
 $= 0 + 1(0.1)$   
 $= 0.1$