Lesson 29 : Section 4.9

Antiderivatives

- The antiderivative, or indefinite integral, "undoes" the derivative.

$$\int \underbrace{f(x)}_{\text{"integrand"}}\, dx = F(x) \text{ means } F'(x) = f(x).$$

- We can check our answers by taking the derivative and making sure we get back the integrand.

Basic Rules

- $\int f(x) + g(x)\, dx = \int f(x)\, dx + \int g(x)\, dx$
- If c is a constant,

$$\int c \cdot f(x)\, dx = c \cdot \int f(x)\, dx.$$

Antiderivatives of Common Functions

"constant of integration"

- $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$ if $n \neq -1$
- $\int \frac{1}{x}\, dx = \ln(|x|) + C$ Notice the absolute value
- $\int e^x\, dx = e^x + C$
- $\int b^x\, dx = \frac{1}{\ln(b)} \cdot b^x + C$
- $\int \sin(x)\, dx = -\cos(x) + C$
- $\int \cos(x)\, dx = \sin(x) + C$
- $\int \sec^2(x)\, dx = \tan(x) + C$
- $\int \csc^2(x)\, dx = -\cot(x) + C$
- $\int \sec(x)\tan(x)\, dx = \sec(x) + C$
- $\int \csc(x)\cot(x)\, dx = -\csc(x) + C$

- $\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x)+C$
- $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x)+C$
- $\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1}(x)+C$

Practice Problems

1. Compute

(a) $\int \left(\frac{2}{1+x^2}+3\right)dx$

(b) $\int \frac{10t^{3/2}+9}{t}\,dt$

(c) $\int \tan(\theta)\left(\cos(\theta)-\sec(\theta)\right)d\theta$

2. Given that

$$f''(x) = 2+5e^x,$$

$f'(0)=1$, and $f(0)=5$, find $f(x)$.