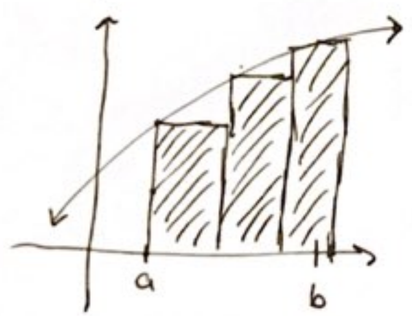
Lesson 30: Section 5.1

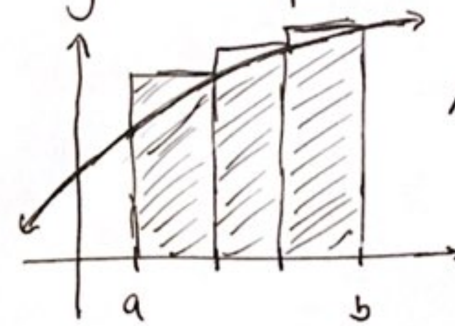
Approximating Area under Curves

- We approximate the (signed) area A under the graph $y = f(x)$ over the interval $[a,b]$ with Riemann sums.
- If we approximate with n rectangles, each rectangle has width $\Delta x = \frac{b-a}{n}$.
- Left Endpoint Sum: Height of rectangles determined by left endpoint.



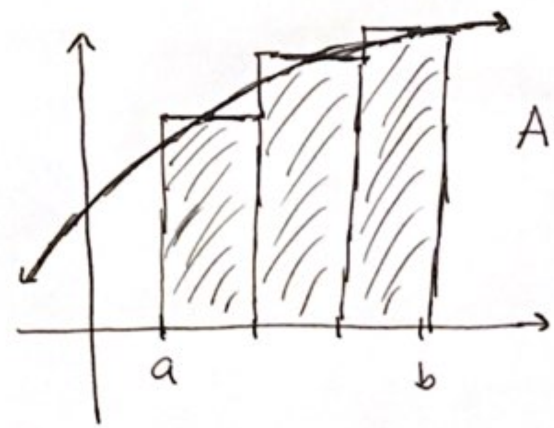
$A \approx$ sum of areas of rectangles

- Right Endpoint Sum: Height of rectangles determined by right endpoint.



$A \approx$ sum of areas of rectangles

- Midpoint Sum: Height of rectangles determined by midpoint



$A \approx$ sum of areas of rectangles

- The approximations for the area under the graph of $f(x)$ get more accurate as we take more and more rectangles.
- We often express sums of many things with sigma notation:

$$\sum_{i=k}^{k+n} f(i) = f(k) + f(k+1) + \cdots + f(k+n)$$

Left Sum: $A \approx \sum_{i=0}^{n-1} f(a + i \cdot \Delta x) \cdot \Delta x$

Right Sum: $A \approx \sum_{i=1}^{n} f(a + i \cdot \Delta x) \cdot \Delta x$

Midpoint Sum: $A \approx \sum_{i=0}^{n-1} f\left(a + \frac{\Delta x}{2} + i \cdot \Delta x\right) \cdot \Delta x$

- Denote

$$\int_a^b f(x)\,dx = \text{exact (signed) area under } f(x) \text{ over the interval } [a,b]$$

Practice Problems

1. Compute the left, right, and midpoint Riemann sums for $f(x) = \sqrt{144 - x^2}$ over the interval $[-12, 12]$ using 6 rectangles. What is the exact area and how does it compare?

2. Write the following sum in sigma notation:

$$\frac{1}{3} + \frac{2}{7} + \frac{3}{11} + \frac{4}{15} + \frac{5}{19} + \frac{6}{23}.$$