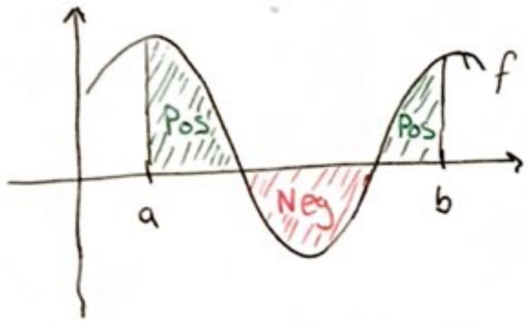
Lesson 31: Section 5.2

Definite Integrals

- Signed area of $y = f(x)$ on $[a,b]$:



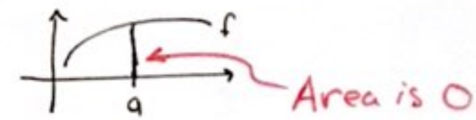
- The signed area can be approximated using Riemann sums.
- The definite integral of $f(x)$ over $[a,b]$ is

$$\int_b^a f(x)\,dx = \lim_{n\to\infty} \left(\begin{array}{c}\text{Riemann sum} \\ \text{with } n \\ \text{rectangles}\end{array}\right).$$

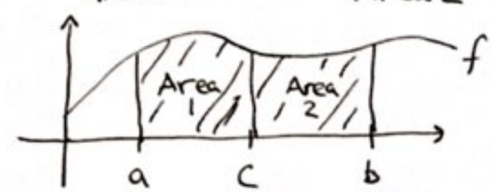
This gives the exact signed area.

Properties of Definite Integrals

- $\int_a^a f(x)\,dx = 0$



- $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
- $\underbrace{\int_a^c f(x)\,dx}_{\text{Area 1}} + \underbrace{\int_c^b f(x)\,dx}_{\text{Area 2}} = \int_a^b f(x)\,dx$



- $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
- $\int_a^b c \cdot f(x)\,dx = c \cdot \int_a^b f(x)\,dx$

Practice Problems

1. Compute

$$\int_{-3}^{100} f(x)\,dx,$$

where

$$f(x) = \begin{cases} \sqrt{9-x^2} & \text{if } -3 \le x < 0 \\ \frac{-3}{5}x + 3 & \text{if } 0 \le x < 50 \\ \frac{27}{10}x - 162 & \text{if } 50 \le x \le 100 \end{cases}$$

2. A function f is odd if $f(-x) = -f(x)$, and f is even if $f(-x) = f(x)$.

(a) If f is odd, compute

$$\int_{-1000}^{1000} f(x)\,dx,$$

and explain how you arrived at your answer.

(b) What can be said about $\int_{-1000}^{1000} f(x)\,dx$ if f is even?