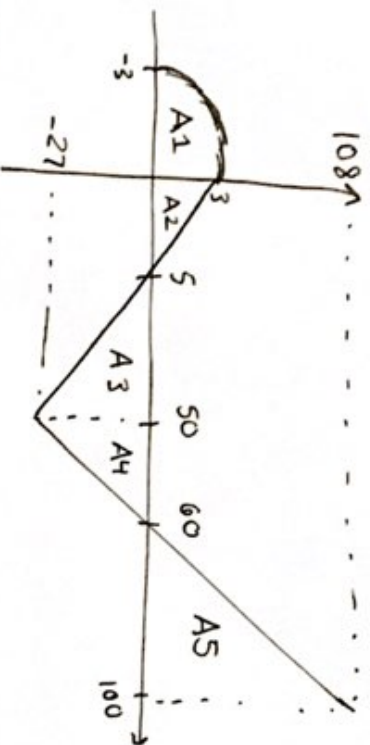


1.

$$f(x) = \begin{cases} \sqrt{9-x^2} & \text{if } -3 \leq x < 6 \\ -\frac{3}{5}x + 3 & \text{if } 6 \leq x \leq 50 \\ \frac{27}{10}x - 162 & \text{if } 50 \leq x \leq 100 \end{cases}$$

A rough sketch of $f(x)$:

$$\begin{aligned} \int_{-3}^{100} f(x) dx &= A1 + A2 - A3 - A4 + A5 \\ &= \frac{1}{4} \pi \cdot 3^2 + \frac{1}{2} \cdot 5 \cdot 3 - \frac{1}{2} \cdot 45 \cdot (27) \\ &\quad - \frac{1}{2} \cdot 10 \cdot (27) + \frac{1}{2} \cdot 40 \cdot 108 \\ &= \frac{9}{4} \pi + 1425 \end{aligned}$$

L31 P3

2.

(a) f odd

$$\int_{-1000}^{1000} f(x) dx = 0 \quad \text{because}$$

$$\begin{aligned} \int_{-1000}^{1000} f(x) dx &= \int_{-1000}^0 f(x) dx + \int_0^{1000} f(x) dx \\ &= \int_0^{1000} f(-x) dx + \int_0^{1000} f(x) dx \\ &= - \int_0^{1000} f(x) dx + \int_0^{1000} f(x) dx \\ &= 0 \end{aligned}$$

Origin is Symmetric

(b) f even

$$\begin{aligned} \int_{-1000}^{1000} f(x) dx &= \int_{-1000}^0 f(x) dx + \int_0^{1000} f(x) dx \\ &= \int_0^{1000} f(x) dx + \int_0^{1000} f(x) dx \\ &= 2 \cdot \int_0^{1000} f(x) dx \end{aligned}$$