Lesson 32: Section 5.3

Fundamental Theorem of Calculus Part 1

- $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$
- Equivalently,
  $$F(x) = \int_a^x f(t)\,dt$$
  is an antiderivative of $f(x)$.

Caution: This only works if x is the upper limit of integration and is by itself.

Example:

$$\frac{d}{dx}\left[\int_{-2x}^{x} f(t)\,dt\right] = \frac{d}{dx}\left[-\int_0^{-2x} f(t)\,dt + \int_0^x f(t)\,dt\right]$$

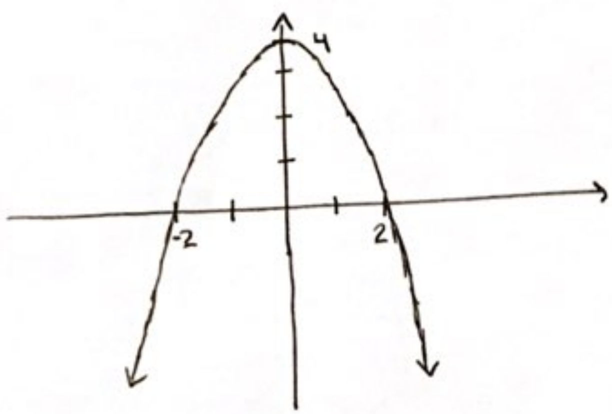
$$= -f(-2x)\cdot\underbrace{(-2)}_{\text{Chain Rule}} + f(x)$$

Fundamental Theorem of Calculus Part 2

- $$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$
  $$= F(b) - F(a),$$
  where F is an antiderivative of f on $[a,b]$
- This allows us to compute the exact signed area under $y = f(x)$ if we can find an antiderivative

Practice Problems

1. Let $g(x) = \int_0^x f(t)\,dt$ where $f(t)$ is pictured below. For what x-values is $g(x)$ increasing/decreasing? Concave up/concave down?



2. If possible, use FTC 2 to evaluate the following integrals. If FTC 2 doesn't apply, explain why.

(a) $\int_{-1}^{1} \frac{-1}{1+x^2}\,dx$

(b) $\int_{-1}^{1} \frac{-1}{x^2}\,dx$